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CR-5-244

OPTICAL SIGNATURES PROGRAM FINAL REPORT

ROCKET PLUME OPTICAL SIGNATURES

October 1972

Prepared by:

McDonnell Douglas Astronautics Company-West
5301 Bolsa Avenue
Huntington Beach, California 92647

for

GENERAL RESEARCH CORPORATION
WASHINGTON OPERATIONS
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This report was prepared by McDonnell Douglas Astronautics Company-West under Subcontract No. 040-71-10 with the General Research Corporation, Washington Operations, for the U.S. Army Advanced Ballistic Missile Defense Agency under Prime Contract No. DAHC60-70-C-0078 MOD 5. Principal Investigator: M. Thomas (714) 896-4212. Optical Signatures Program Manager: T. M. Zakrzewski, (703) 524-7206, Extension 272.

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PREFACE

This document is the final report on the "Rocket Plume Optical Signature Program." This work was performed for General Research Corporation, 1501 Wilson Boulevard, Suite 700, Arlington, Virgnia 22209, under Contract 040-71-10. This contract covered the period from 20 August 1971 to 30 September 1972.

The effort described herein was accomplished by the following study personnel: S. S. Cherry, M. Thomas, and R. L. Younkin. This report was approved by H. Hurwicz, Chief Advance Technology Engineer, Aero/Thermodynamics and Nuclear Effects Research and Development, Advance Systems and Technology, McDonnell Douglas Astronautics Company-West.

ABSTRACT

As part of the Optical Signatures Program, McDonnell Douglas Astronautics Company-West has developed the initial working model of a code to describe the gross features of rocket-plume radiation for altitudes above 75 n mi. The main effort is the construction of a scheme for integration of an arbitrary function through an arbitrary axisymmetric rocket plume, with any specified look angle, plume direction, and vehicle velocity direction. Radiances are presented as integrated values in a specified spectral band. The equations used and a printout of the code and of a sample application are included.

CONTENTS

FIGURES

TABLES

Section 1
INTRODUCTION AND SUMMARY

A code has been developed to describe the gross features of rocket-plume radiation for altitudes above 75 km (P2170-FLAME). The effort was intended to provide an initial working model, consistent with available technology. Theoretical studies were not intended to be conducted in any of the disciplines required. The intent was, however, to provide a suitable framework for the description of a plume with sufficient flexibility to incorporate the results of future research.

This objective was met by considering the two most important far-field radiative mechanisms, particle emission and atmospheric excitation, in their present state of understanding. With these major elements of plume radiation included even in crude form, future program modification to include revised theories should not require complete program revision. The main effort which has been pursued in FLAME-code development is the construction of a scheme for integration of an arbitrary function through an arbitrary axisymmetric rocket plume, with any specified look angle, plume direction, and vehicle velocity direction. The assumption of flow symmetry about the plume axis may prove poor for late-time plumes, but to include a complete three-dimensional capability would have burdened the code's development to preclude an operational version at the end of this contract. The switch to 3-D is not difficult at a later time and will involve obtaining flow correlations as a function of the azimuthal angle, as well as r and θ, and integrating over four quadrants instead of two.

Radiances are presented in the current code as integrated values in a specified spectral band. Spectral intensities are readily available for the particles, but only a band theory is available for the atmospheric excited species, due primarily to the species cooling characteristics. Again, a switch to spectral curves is possible with ease at a later time when a more general cooling theory evolves.

A schematic of the FLAME code is presented in Figure 1-1. Radiative cooling of both the hot particles in multiphase plumes and the excited molecular radiators is included. The four boxes feeding into the FLAME code represent the main areas programmed during this contract. The use of a simplified flow model reduces both computer storage requirements and run time, and allows a more extensive checkout of the radiative mechanisms. More sophisticated flow models exist and could be substituted at a later time.

Input to the code includes simple engine parameters (Section 3, Table 3-1). Output is a plot of lines of constant radiance in the plume as viewed in the specified direction. A typical output from the code is shown in Figure 1-2 for a broadside look at a solid propellant plume in the absence of atmospheric excitation. The radiance is plotted in a plane perpendicular to the direction of view. The plume of Figure 1-2 is shown again in Figure 1-3 where the observer is at a 45-degree angle to the plume axis. Radiance from the same engine at low altitudes, including the effect of atmospheric excitation of the plume, is shown in Figure 1-4. The viewer angle, plume direction, and free stream velocity direction are all completely arbitrary. (Section 3, Figures 3-1 and 3-2.)

Only one-half the plume is treated by a single FLAME run. The remaining half can be obtained by redefining the azimuthal angles as shown in Section 3, Figure 3-2. It is not always necessary to generate both plume halves because the plume radiance is axisymmetric under certain conditions.

The radiative models used tend to underpredict the radiation. For atmospheric excitation, only single collisions are considered and the Boltzmann distribution of excited rotational states is assumed. Persistance of the plume radiance due to pumping by thermal (rather than pure kinetic) collisions with the atmosphere is not treated. For particle radiation, scattering of Earthshine is not treated.

Section 2 summarizes the theory used. Sections 3 and 4 detail the workings of the FLAME code, describing the equations used.

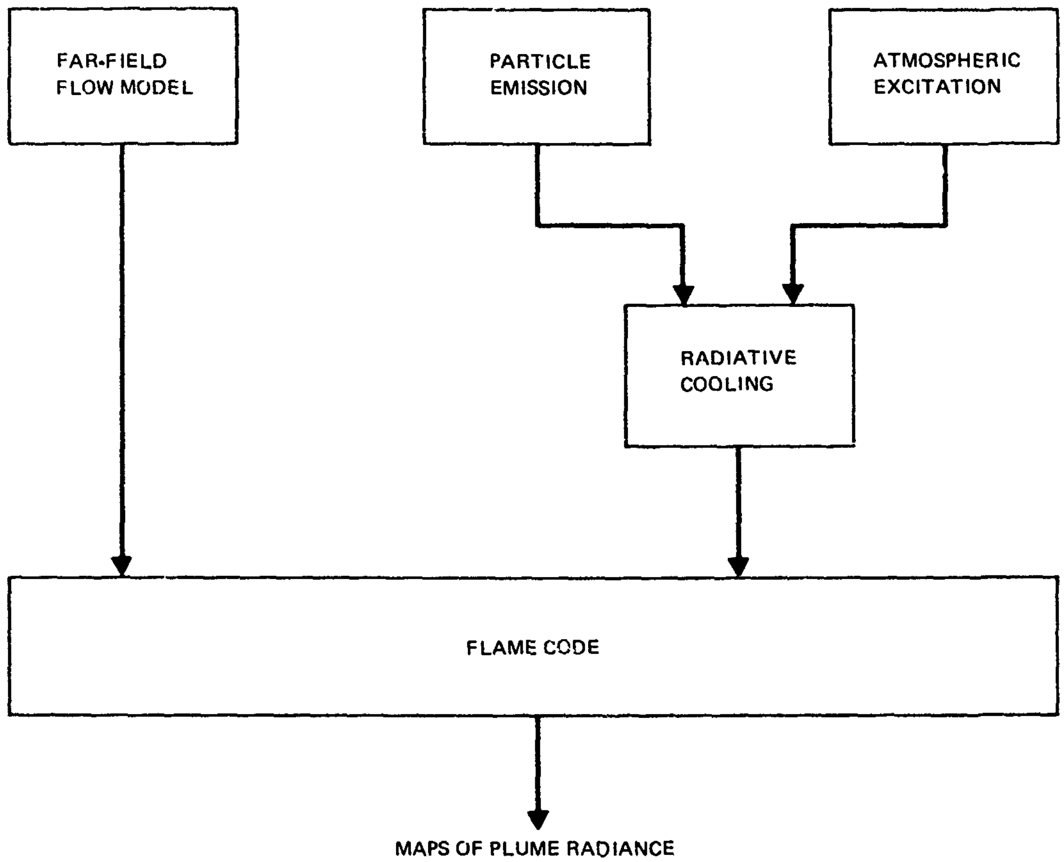

Figure 1-1. Components and Flow of FLAME Code

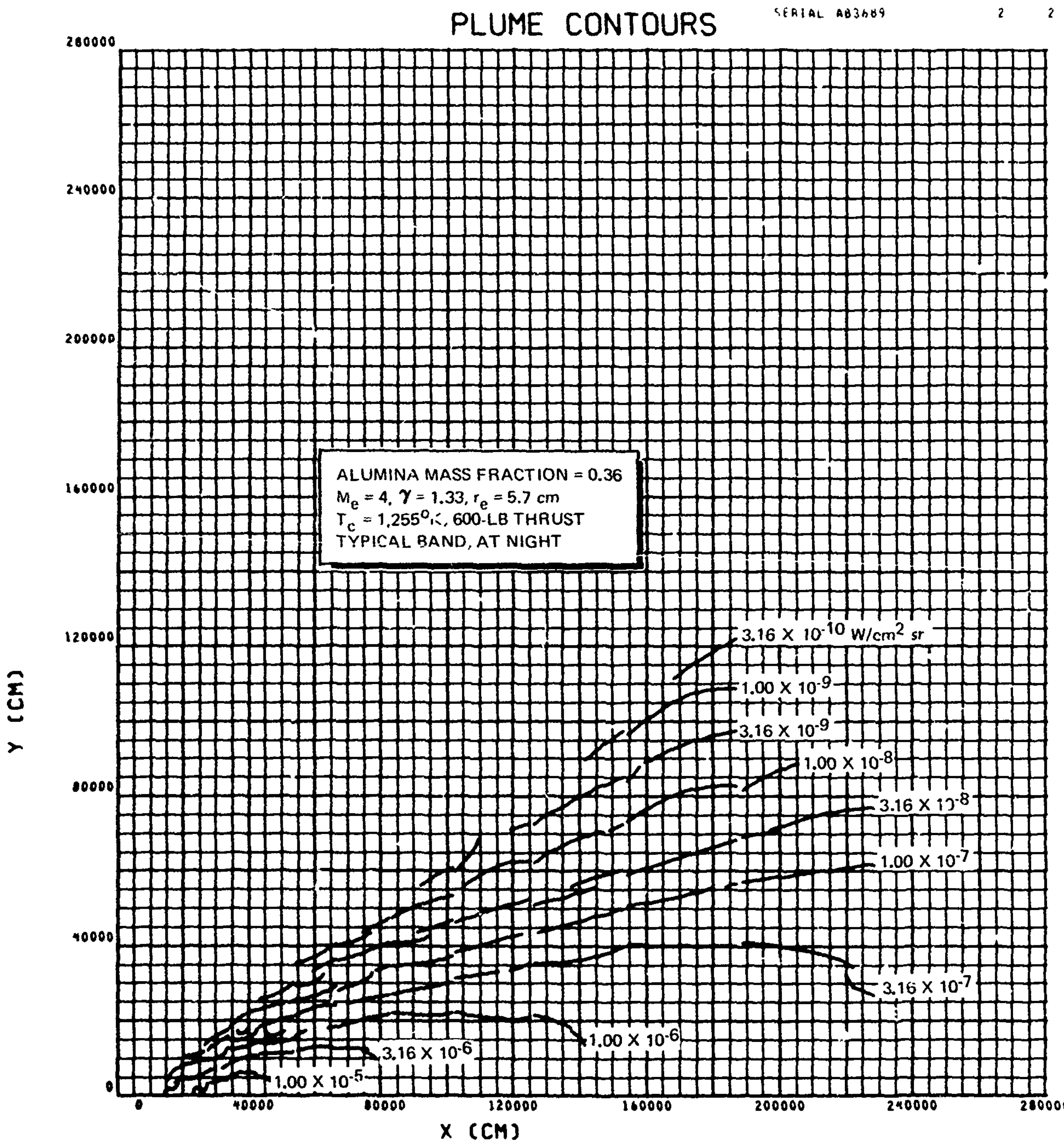

Figure 1-2. Isointensity Lines for Braodside Look at Exoatmospheric Plume of Solid Propellant Rocket

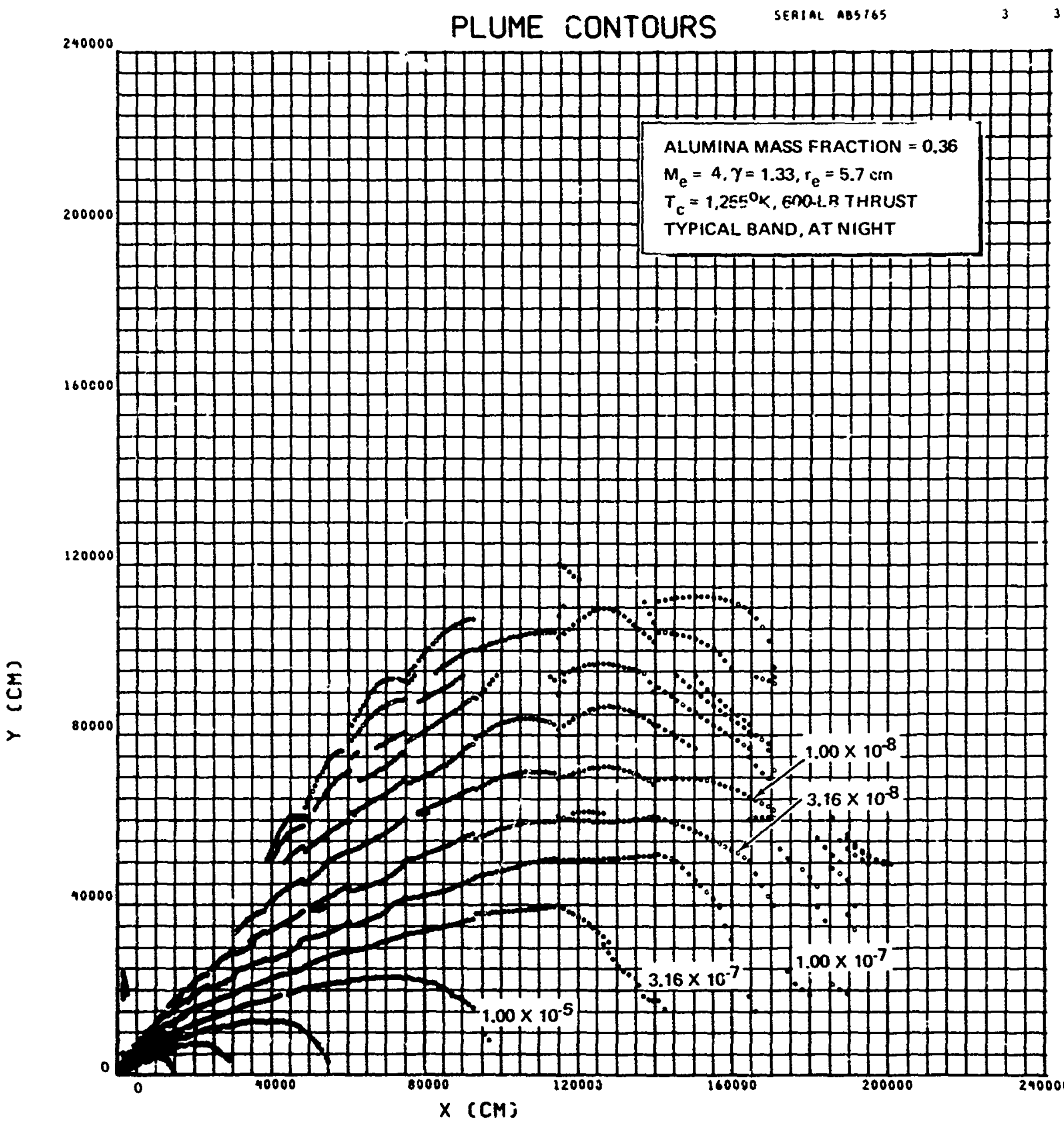

Figure 1-3. Isointensity Lines for Look at 45° from Plume Axis of Solid Propellant Rocket

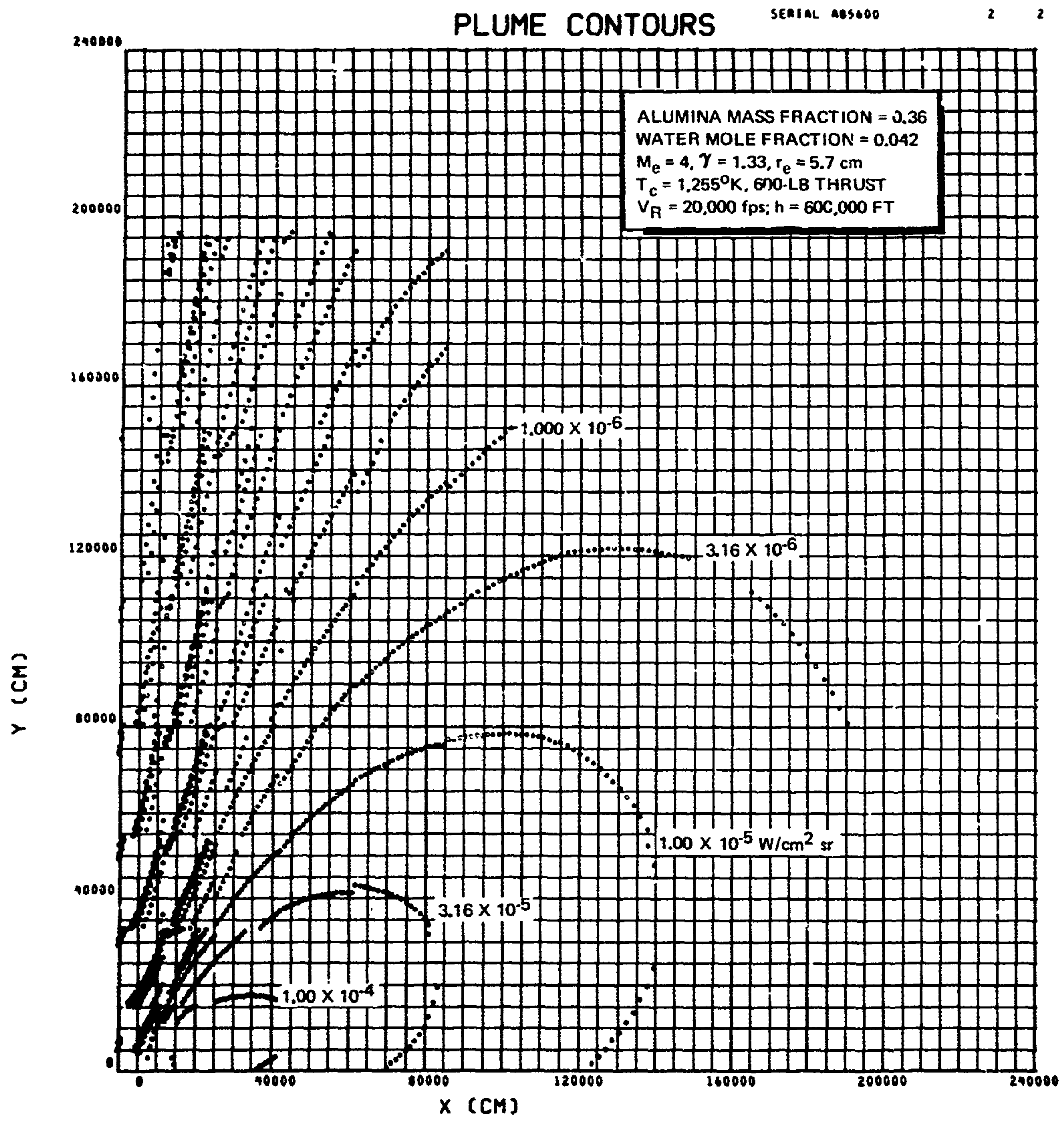

Figure 1-4. Isointensity Lines for Broadside Look at Particle and Excited Water Radiation from Solid Propellant Rocket

Section 2
THEORY

The major elements of the theory used as the basis for the FLAME code is outlined in this section. The approximate model of the far-field multiphase plume is described. For two-phase plumes, closed-form expressions for particle concentration as a function of plume coordinates and particle size are presented. The radiative mechanism for the particle cloud is simply emission, with an effective minimum temperature defined by solar illumination or scattering of Earthshine; it need not be elaborated on here. Gas radiation is considered to be caused by excitation of water rotational spectrum by collision with the atmosphere; this theory is outlined in subsection 2.3. The model can be generalized to include other radiating rotors.

The approaches used were based on existing theoretical work. No attempt has been made to derive new theoretical expressions or include radiative mechanisms not yet understood in the current FLAME. The program is flexible enough, however, to allow the inclusion of other mechanisms at some future time.

2.1 PLUME GAS FLOW

Closed-form approximation to the plume gas is relied upon for flow applicable to the far-field plume. Atmospheric mixing and the resulting plume slow down have not been addressed. Atmospheric excitation of the unaltered plume is considered.

Closed-form solutions have been developed for the density profiles as a function of engine operating conditions. It appears that all properties of the far-field gaseous plume can be thus treated. The computer program will respond to input of engine thrust (or chamber pressure), chamber temperature, expansion ratio (ϵ), γ (or exit mach number, M_e), and nozzle lip angle by generating a complete history of the rocket plume.

To predict transition to free-molecular-flow, a relationship was derived to express the Knudsen number (K_n) variation in a bipropellant rocket engine plume as a function of the polar coordinates (r, θ) and basic engine parameters F (thrust), C_F (thrust coefficient), T_c (combustion temperature), M_e (exit Mach number) and (ratio of specific heats). The gas viscosity, μ, was assumed to vary as $T^{1/2}$. The engine chamber pressure, P_c, could be introduced through the basic thrust equation:

$$F = P_c A_t C_F$$

where: A_t = Throat Area

These derivations used Roberts' approximation (Reference 1) for the plume gas density which indicated that the density was inversely proportional to r^2 and proportional to $\cos\theta$ raised to the power $\gamma(\gamma - 1)\, M_e^2-2$.

The derived relationship is shown in Figure 2-1 as a function of M_e, γ, and ϵ (engine expansion area ratio). This information may be employed to determine the shape of the surface (r, θ) required to achieve a given K_n. It is noted that r increases with increasing γ for specified values of T_c, K_n, F, and θ. Indeed, this relationship indicates that $r \rightarrow 0$ as $\theta \rightarrow 90$ degrees, i.e., that transition occurs at the nozzle lip where, theoretically, there is an infinite pressure gradient as the flow undergoes a Prandtl-Meyer expansion.

Figure 2-2 presents the variation of plume density on a spherical cap (r = constant) as a function of θ and M_e with $\gamma = 1.4$. As shown, the density has been normalized by its value on the engine centerline, i.e., $\theta = 0$. When $M_e = 2.312$, the density distribution is linear with $\cos\theta$ and becomes increasingly dependent on $\cos\theta$ as M_e increases, e.g., for $M_e = 5.0$, $\rho \sim (\cos\theta)^{12}$.

2.2 PARTICLES

A combined nozzle and plume flowfield calculation was performed for an aluminized solid propellant motor to obtain correlations of particle characteristics in the near field. Closed-form solutions, analogous to those obtained for gas-phase-only plumes were not possible due to the nonequilibrium nature of the expansion involving momentum (velocity) and energy (temperature) differences between the phases.

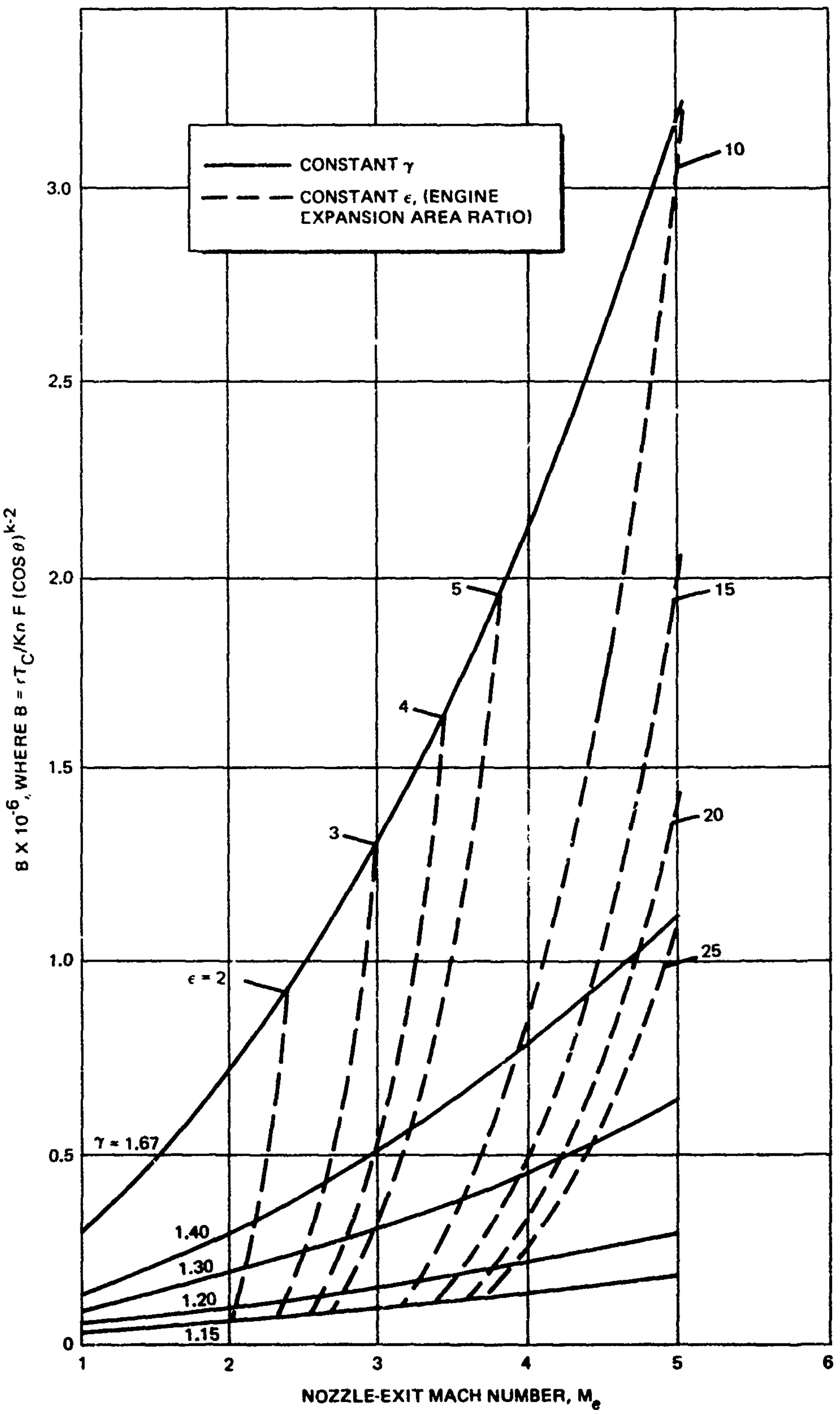

Figure 2-1. Knudsen Number Variation with Engine Conditions

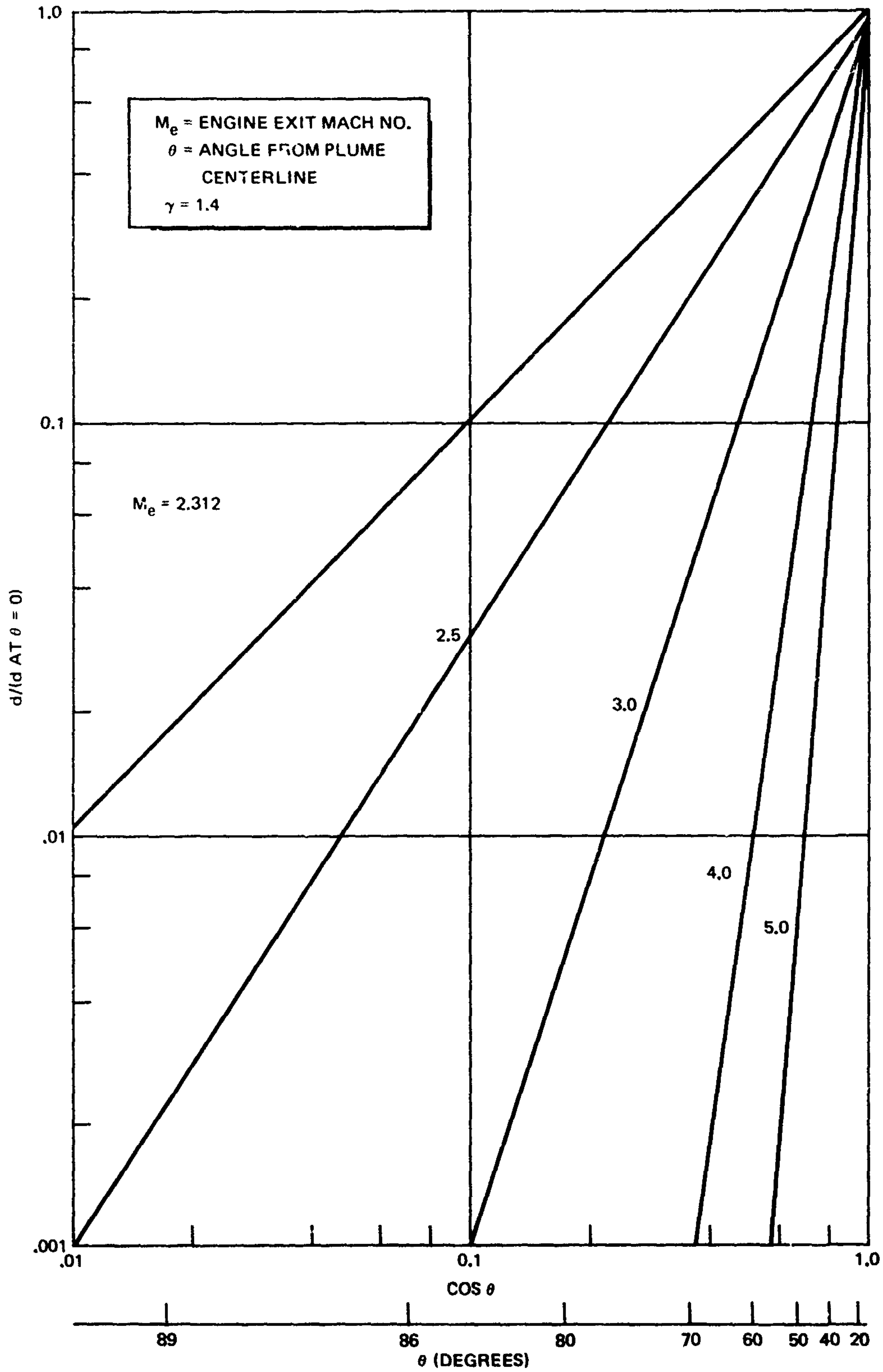

Figure 2-2. Plume Density Distribution on Spherical Cap (r = constant)

Correlations were obtained which related particle velocity, temperature, density, and limiting streamline direction to the seven particle-size groups considered. An expression was derived which related the particle Knudsen number to plume coordinates and motor operating conditions. The primary use of this expression was to determine the extent of the particle continuum region.

Initial estimates of particle emissivity were made, based primarily on existing calculations for alumina. The details of these analyses are presented in later subsections.

2.2.1 Two-Phase Flowfield Particle Correlations

The computer programs described in References 2 and 3 were employed to calculate the nozzle and plume flowfields for an aluminized solid-propellant motor with the following characteristics:

Thrust = 15,000 lb_f
Chamber pressure = 550 psia
Expansion area ratio = 30.8
Aluminum weight content = 16 percent

A total of seven aluminum-oxide particle groups were utilized with particle radii, ρ_i, ranging from 0.6 to 4.7 μm. The corresponding number count versus ρ_i was obtained from Reference 4.

The computed output contained particle velocity, temperature, flow directions, and nondimensional number density for each group at flowfield mesh points within the plume. This information was processed to extract certain particle asymptotic characteristics, namely; velocity, temperature, and flow direction along the particle-limiting streamlines. (The limiting streamline is defined as the boundary outside of which a given particle size will not be present.) It was found that these streamlines became straight lines after the particles had traversed a relatively short distance due to the rapid decay of interphase momentum and energy exchange.

The values of particle temperature, T_i, flow angle, θ_i, and velocity, $V(\rho_i, \theta)$, on the limiting streamlines are shown in Figure 2-3 as a function of ρ_i. It is noted that particles larger than 4μ are still undergoing the liquid-solid phase transition at the aluminum-oxide melting temperature of 4,170 R and that these particles are contained within a 23.5-degree half-angle cone. Only forced convection heat transfer between the particles and the gas is considered in the computer programs and, therefore, a further cooling by radiation must be considered separately.

The plume computer output was employed to obtain the particle velocities for the seven discrete-size groups considered. The correlation was obtained in the form:

$$V(\rho_i, \theta) = [a_i + b_i \exp(-c_i R/r_t)] (\cos\theta)^{n_i}, \quad 0 \le \theta \le \theta_{\rho_i} \tag{2-1}$$

where: R, θ = Polar coordinates in plume

r_t = Nozzle throat radius

θ_{ρ_i} = Particle limiting streamline inclination (Reference 1)

Table 4-1 in Section 4 shows the correlation coefficients. The a_i coefficients decrease with increasing particle diameter which indicates that the larger particles achieve a lower limiting velocity as $R/r_t \gg 1$. In addition, the n_i coefficients were all negative which indicates that the particle velocities increase off-axis on a given spherical cap. This effect is due to the higher particle concentration on-axis which has caused more interphase momentum transfer from the gas to the particles, i.e., on a per particle basis the gas is less about to "drag" the particle. The lower off-axis particle density allows the gas to "drag" the particles to higher velocities.

The particle density correlation was redone specifically for the off-axis dependency:

$$\frac{d\rho_i}{d\rho_o} = a_i \left(\frac{r_t}{R}\right)^{n_i} (\cos\theta)^{b_i}, \quad 0 \le \theta \le \theta_{\rho_i} \tag{2-2}$$

The correlation coefficients are given in subsection 4.5.1.4.

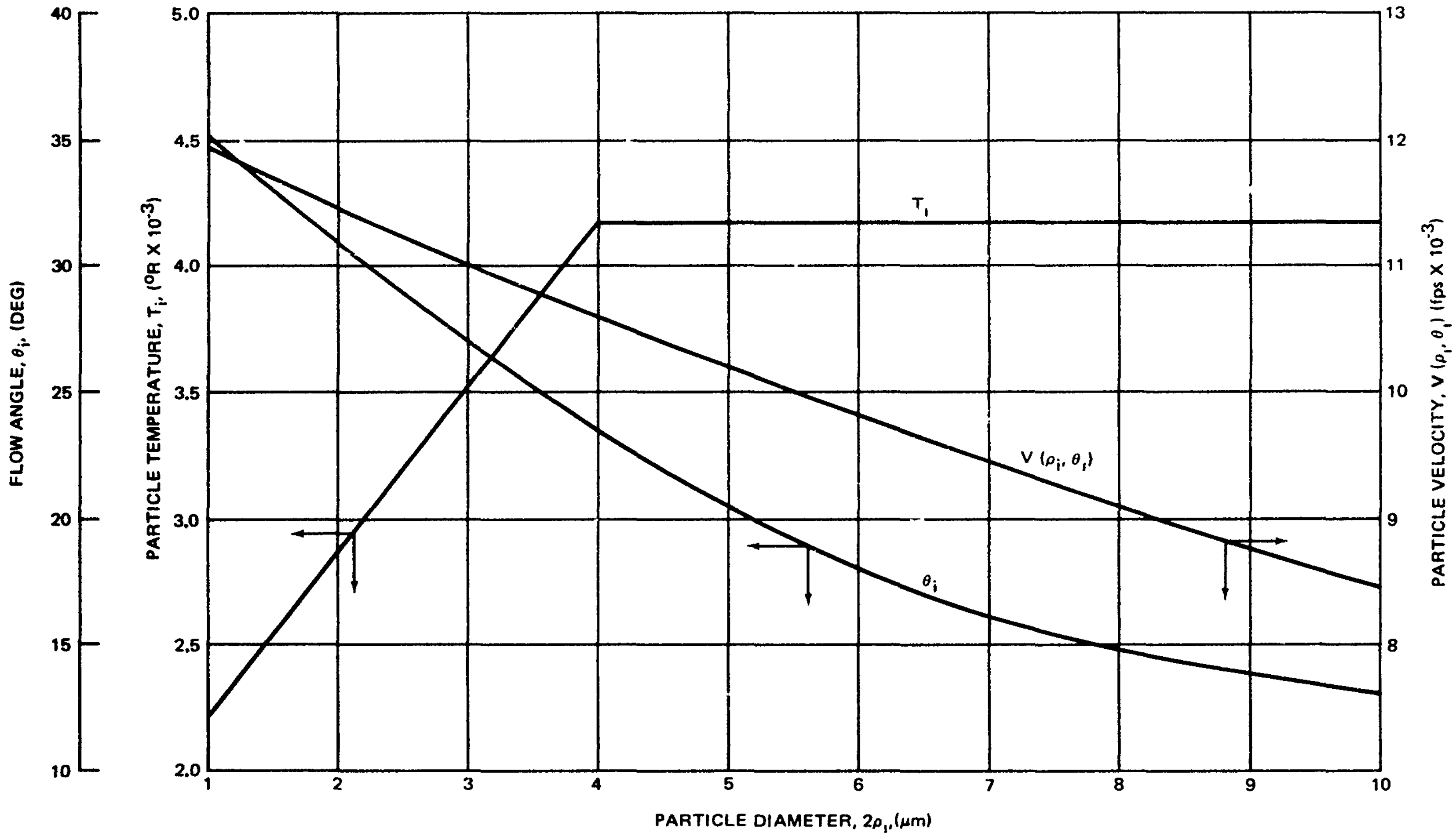

Figure 2-3. Particle Temperature, Flow Angle, and Velocity on Limiting Streamline

Aside for the largest particle size considered, the correlation showed that $b_i > 0$, i.e., the particle density decreased off-axis.

The two-phase computer results were analyzed to obtain a correlation of the gas phase mass flux on the engine centerline with the result being:

$$\left(V \frac{d}{d_c}\right)_{\theta = 0} = a \left(\frac{R}{r_t}\right)^{-2.7443} \tag{2-3}$$

where d_c = chamber density

The above result indicates that the mass flux is decaying more rapidly than inverse square with distance for the conditions employed. Additional computer runs should be performed for lower value of the solids loading in the exhaust products to determine if the gas phase mass flux will approach a $\left(\frac{R}{r_t}\right)^{-2}$ dependency.

Following Reference 5, the off-axis gas phase mass flux was assumed to be:

$$\frac{\left(V \frac{d}{d_c}\right)_{\theta}}{\left(V \frac{d}{d_c}\right)_{\theta = 0}} = \exp\left[-\lambda^2 (1 - \cos c\theta)^2\right] \tag{2-4}$$

where $c = \dfrac{90°}{\theta_e + (\nu_{max} - \nu_e)}$

and θ_e = Nozzle lip angle at exit

ν_{max} = Maximum Prandtl-Meyer angle

ν_e = Prandtl-Meyer angle at exit Mach number

An expression was derived which relates the particle Knudsen number, Kn_{p_i}, to engine operating conditions, location, and particle size for a two-phase plume. This expression was obtained by defining Kn_{p_i} as the ratio of the relative Mach number and Reynolds number, i.e.,:

$$Kn_{p_i} = \frac{Mr_i}{Re_i} = \frac{\left|V-V_{p_i}\right| / \sqrt{g_c \gamma R T}}{d \left|V-V_{p_i}\right| 2\rho_i / \mu} \tag{2-5}$$

where:

- V = Gas velocity
- V_{p_i} = Velocity of the ith particle
- g_c = Conversion factor (= 32.2 [lb_m/lb_f]). (ft/sec^2)
- γ = Gas ratio of specific heats
- R = Gas constant
- T = Gas static temperature
- d = Gas density
- ρ_i = Radius of the ith particle
- μ = Gas viscosity

Roberts' solution (Reference 1) was employed to relate d to location in the plume and to certain engine operating parameters; the viscosity was assumed to vary as $\sqrt{T}$; and the isentropic relation was used to eliminate T. With these substitutions Equation 1 becomes:

$$Kn_{p_i} = \left[\frac{12 \times 46.6 \times 10^{10} \sqrt{R_o/g_c}\; T_c}{P_c \gamma^{3/2} (\gamma-1)\, M_e^2 \left(1 + \frac{\gamma-1}{2} M_e^2\right)} - \frac{1}{\gamma-1} \right] \frac{(r/r_e)^2}{\rho_i \cos^{k-2}\theta}$$

where:

- r_e = Nozzle exit radius
- R_o = Universal gas constant (= 1545 ft. lb_f/mole ^oR)
- T_c = Combustion temperature
- P_c = Chamber pressure
- M_e = Nozzle exit mach number
- r, θ = Polar coordinates of point in plume
- k = Hypersonic similarity parameter (= $\gamma(\gamma-1)m^2$)

The above equation indicates that, for given engine operating conditions and plume coordinates, the smaller particles will achieve the higher Knudsen number, i.e., they will "transition" sooner. Numerical calculations for a 1μ radius particle and typical engine operating conditions indicates that Kn_{p_i} reaches unity very close to the nozzle exit plane. By comparison a gas-phase plume achieves a unity Knudsen number only at large distances from the exit plane.

2.2.2 First Estimate of Emissivity of Alumina Particles

The Mie theory for scattering of electromagnetic waves by dielectric spheres forms the basis for theoretical determination of particle emissivity. The theory actually computes the scattering and absorption efficiency factors, $Q_s(\alpha)$ and $Q_a(\alpha)$, defined by

$$Q = \sigma/\sigma_g$$

where σ_g is the geometrical cross section and σ the actual cross section. α is the size parameter $2\pi a/\lambda$, where a is the particle radius. The values obtained for Q are determined by the values of the real and imaginary parts of the index of refraction n, related by

$$n = n_1 - in_2$$

Since $n = n(\lambda, T)$, a more accurate functional dependence of Q would be $Q=Q(\lambda, a, T)$.

Plass (References 6 and 7) has computed efficiency factors for alumina spheres of 0.1 to 10μ for a range of wavelengths 0.5 to 10μ. Values of n_1 and n_2 used in this wavelength range were based upon measurements made with large sapphire crystals. They indicates $n_2/n_1 \ll 1$.

For determination of the emissivity, ϵ, Plass used an equation originally derived for the emissivity of a semi-infinite homogenous slab, with

$$\epsilon = 2.3\sqrt{\sigma_a/\sigma_s}$$

Thus the following values of ϵ are obtained from the following table:

λ	r	ε
2μ	1.1μ	0.005
	5.1	0.006
	9.9	0.018
5μ	1.1	0.054
	5.1	0.089
	9.9	0.115

(Figures for $\lambda = 10\mu$ are not immediately available).

It is to be noted that in the range $10<\lambda<20\mu$, values of n_2 increase by more than an order of magnitude (Brannon and Goldstein, Reference 8) so ϵ would be expected to be larger there, aside from size effects.

The results of Plass have been questioned by Morizumi and Carpenter (Reference 9) on the grounds that measurements of plume emissivities of rocket exhaust indicate larger values for particle emissivities. This conclusion is rationalized on the grounds that absorbing properties of rocket alumina particles are higher than those of sapphire, due to the polycrystalline properties of the former. They conclude that emissivity of alumina particles in rocket exhausts lies between 0.1 and 0.3.

It is concluded that conflicting and insufficient data available at this time precludes a definitive answer for the emissivity in the infrared. For provisional values, the following are suggested, using linear interpolation and extrapolation to obtain values at other sizes or wavelengths.

λ	r	ϵ	λ	r	ϵ	λ	r	ϵ
5μ	1μ	0.05	10μ	1μ	0.1	20μ	1μ	0.2
	5	0.09		5μ	0.15		5	0.25
	10	0.12		10μ	0.20		10	0.30

An approximate analytic correlation of the above data for Al_2O_3 emissivity was found. The correlation is shown in Figure 2-4, and is adequate, considering the coarseness of the available data. The emissivity correlation is of the form

$$\epsilon_\lambda(r) = (\lambda + 2.5)(.00689 + 11.32\rho - 2220\rho^2) \qquad (2\text{-}6)$$

for λ in microns and ρ (particle radius) in cm.

2.3 ATMOSPHERIC EXCITATION

Atmospheric excitation of rocket-plume species by collisional high relative velocities has been identified as a strong source of plume radiation. A detailed analysis of the problem was presented in Reference 10 and summarized in Reference 11. The unique feature of this analysis is the inclusion of the time-dependent radiative decay in the treatment of the excited species. Thus, there is a finite time required before the plume can generate radiation by this mechanism.

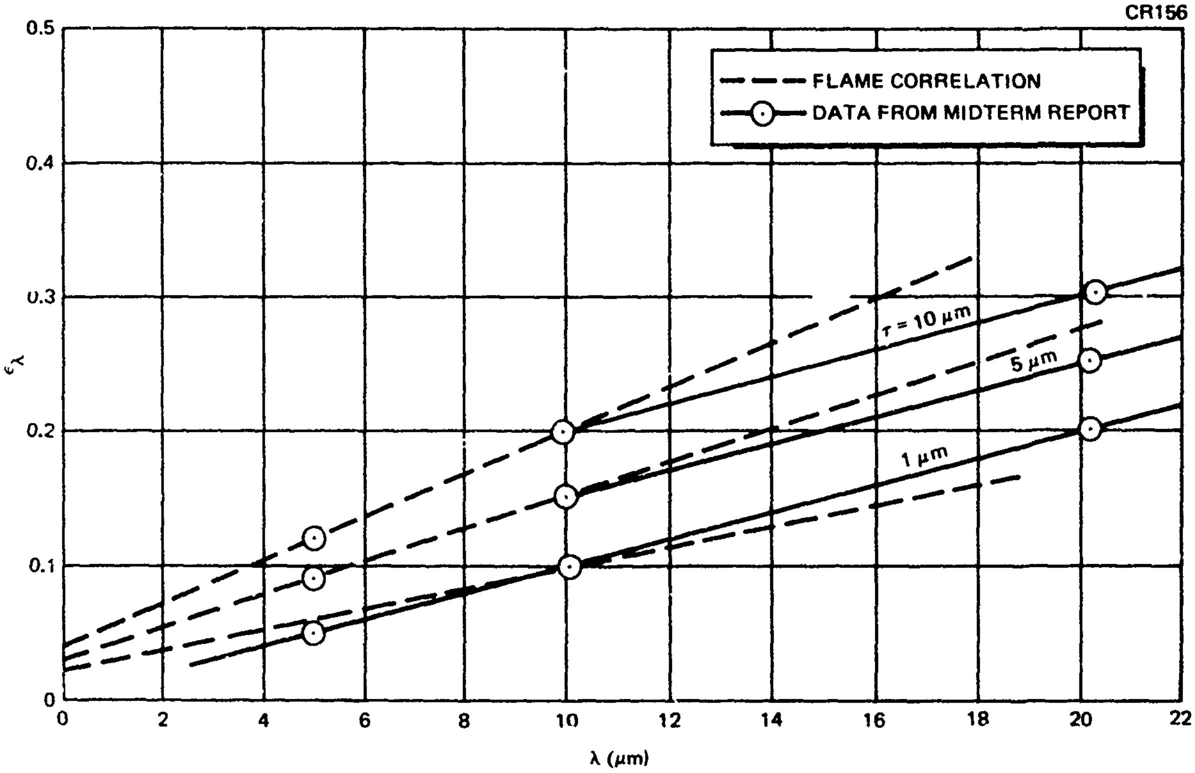

Figure 2-4. Alumina Particle Emissivity

Detailed calculations of trajectories of the colliding species, averaged over initial orientations, were done using Program P2160. Figure 2-5 shows the resulting rotational temperatures induced as a function of impact parameter and V_∞. Also shown on Figure 2-5 is the linear variation found for T_m/V_∞ as a function of V_∞ (T_m is the maximum rotational temperature for any impact parameter).

The effective collision cross section (S) to use with T_m is found by requiring

$$T_m S = 2\pi \int_0^\infty T(b)\, b\, db$$

Calculated values of S are shown in Figure 2-6. For the current version of FLAME, S will be assumed to be constant ($=0.7\times10^{-5}$ cm^2).

The atmospheric interaction model used in FLAME follows that of Reference 10 namely:

A. For the wavelength band of interest, the total energy (E_T) and radiance (j) per molecule are calculated as a function of T.

B. Radiative decay times are calculated as a function of T_m.

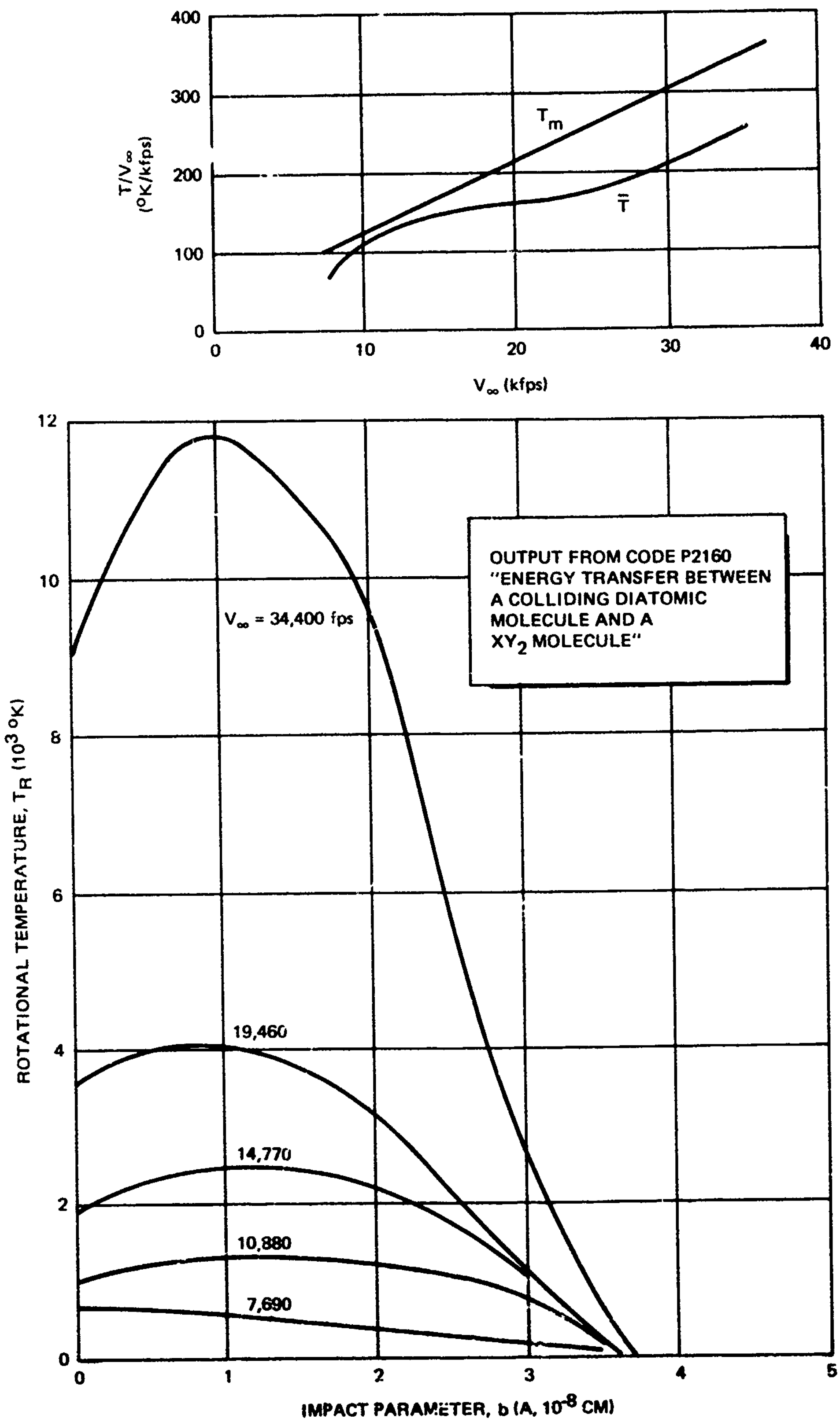

Figure 2-5. Water Rotational Energy from Single Collision

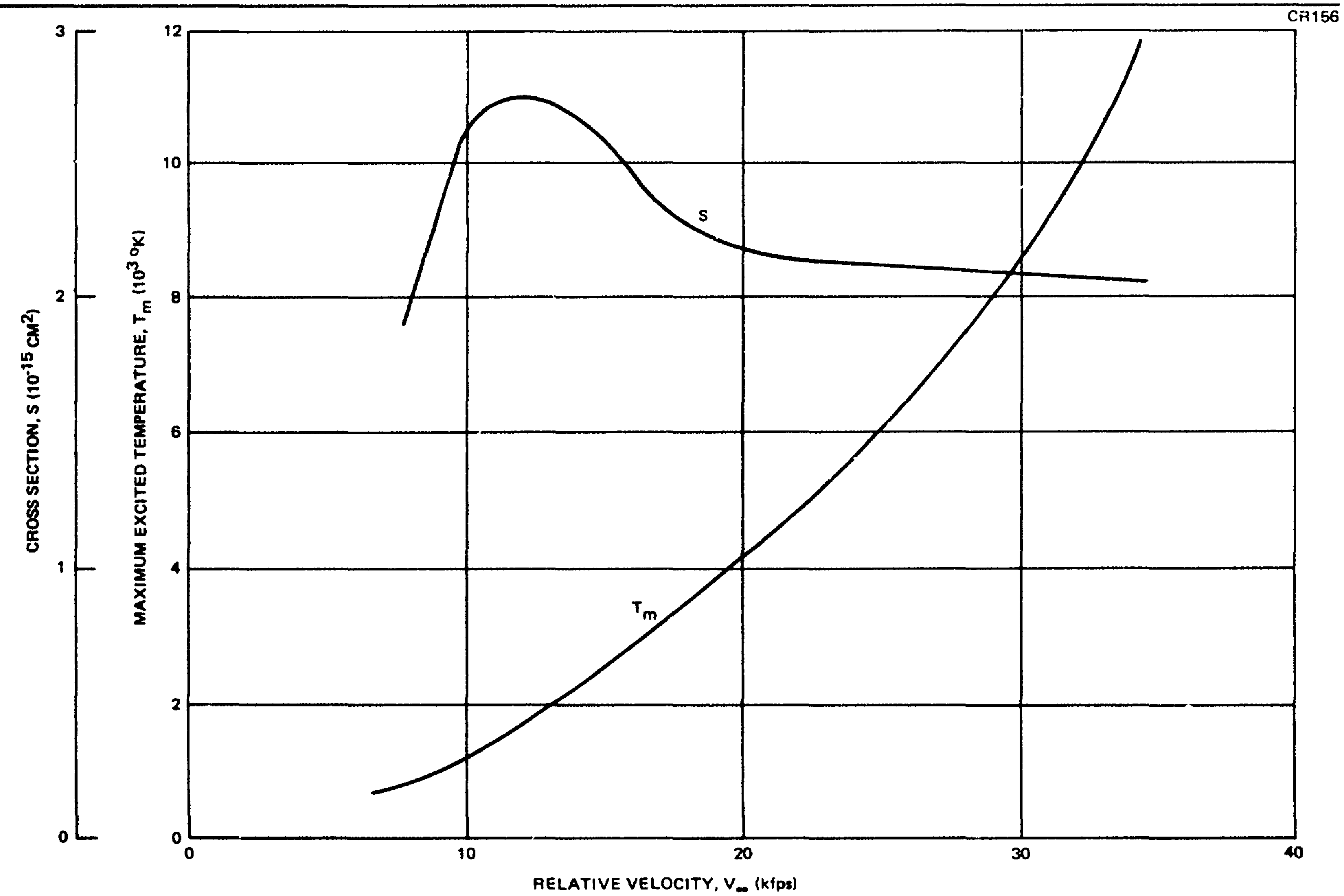

Figure 2-6. Rotational Excitation Parameters for H_2O

C. The time of flight to a position in the plume is compared to the decay time so that an average radiance per excited molecule can be found.

D. The fraction of molecules that are excited are calculated assuming initially cold plume species.

E. No mechanism currently is programmed to account for plume/atmosphere mixing, but plans are to add this later. Currently if a molecule undergoes multiple collisions in the atmosphere, the resultant radiance is found by simple addition.

F. The radiance decay of an excited species is assumed to be linear in time.

G. Data currently in program is applicable to a single species, H_2O.

Further details of the analysis are available in Reference 10.

The analysis of Reference 10 was changed in one way. Since the radiative model used assumes single collisions, and the plume may be viewed from long distance, an attenuation factor (A_t) must be introduced to account for depletion of the supply of uncollided molecules as the gas expands far from the nozzle. Assuming each collision removes that molecule from further consideration, the reduction of the number of available molecules from N_o (initial value) to N (at point r in plume) is given by

$$\frac{N}{N_o} = \exp(-A_t) \tag{2-7}$$

where

$$A_t = S \int_o^r n_\infty \, ds,$$

where S is the collision cross section and n_∞ is the atmospheric density.

A correlation was obtained for the spectral linear absorption coefficient of the H_2O rotational spectrum. The results are that

$$k_\lambda = \log^{-1}(y) \quad \text{in cm}^{-1} \tag{2-8}$$

where

$$y = A \frac{(\omega - \omega_o)^2}{\omega} + y_o$$

$$\omega_o = 10.6 \sqrt{T}, \text{ T in } ^oK$$

$$y_o = \frac{279.96}{\sqrt{T} + 50.031} - 3.208$$

$$A = \left[1.030 - \frac{105.582}{\sqrt{T} - 2.896}\right] \times 10^{-4}$$

$$\omega = 10,000/\lambda, \ \lambda \text{ in } \mu m$$

The data correlated are shown in Figure 2-7, where the results of the correlation are shown for temperatures of 600°K and 1,800°K.

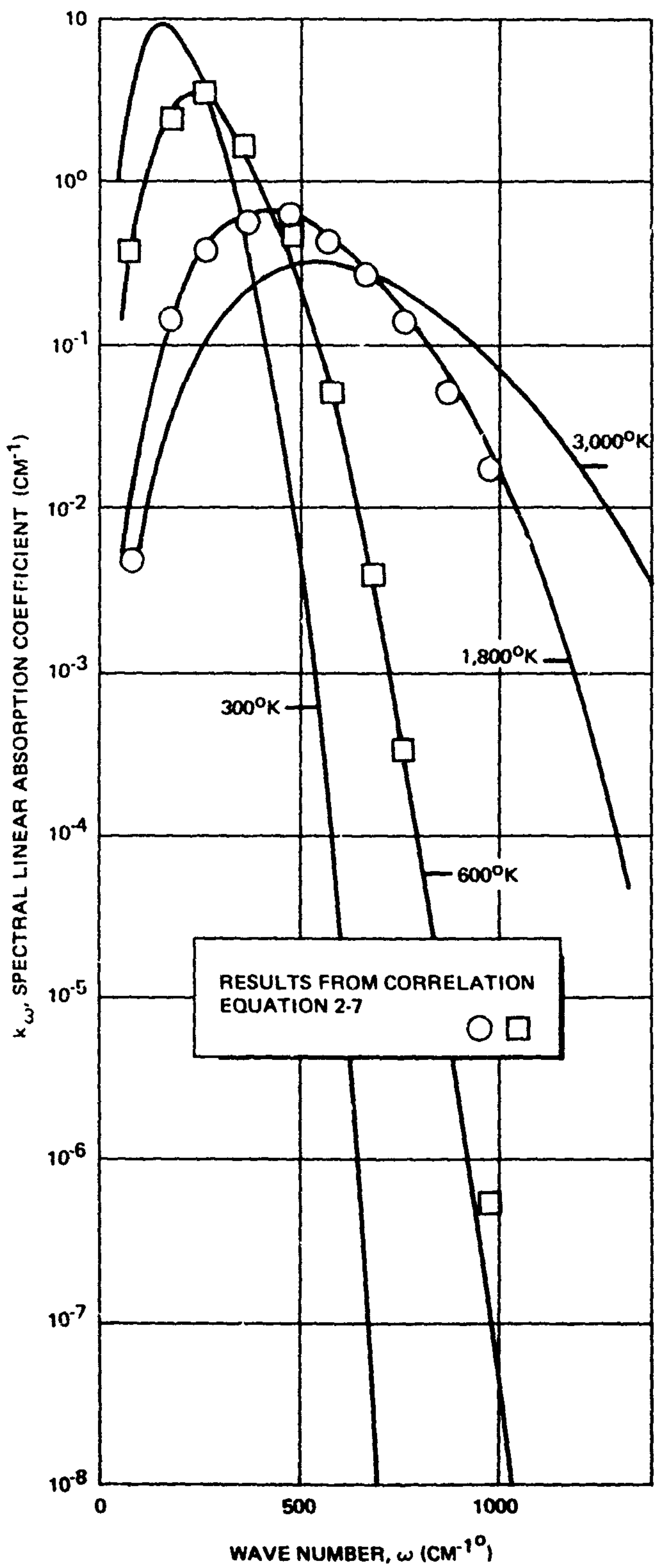

Figure 2-7. Absorption Coefficient from Water Rotation at One Atmosphere Pressure

Section 3
FLAME CODE

The following subsections deal with the logic underlying the FLAME code as it is now configured. Subsection 3.1 describes the logic that went into the choice of coordinate systems, and displays the geometric model used. Subsection 3.2 presents the flow charts used as the basis for FLAME.

3.1 GEOMETRY CONSIDERATIONS

The rocket plume is an axisymmetric body about the z-axis, as illustrated in Figure 3-1. Integration through the plume is carried out through the conical volume in the first and second quadrants defined by the polar coordinates θ_m and r_m. The viewing angle is specified completely by the angle α with respect to the x-axis in the xz plane.

We want to integrate emittance along families of lines of sight, such as s,

$$\int E(r, \theta)\, ds$$

We can express ds in terms of either cartesian or spherical coordinates,

$$ds = dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2} \qquad (3\text{-}1)$$

or

$$ds = dr \sqrt{1 + r^2 \left(\frac{d\phi}{dr}\right)^2 + r^2 \sin^2\phi \left(\frac{d\theta}{ar}\right)^2} \qquad (3\text{-}2)$$

Since emittance is a function of r, θ along, equation (3-2) is more inviting to use, if it is practical. The following analysis shows, however, that cartesian coordinates will probably result in less computation time, and should be used.

The conical segment in Figure 3-1 has equation

$$x^2 + y^2 = z^2 \tan^2 \theta_1 \qquad (3\text{-}3)$$

The family of lines along direction of observation, α, have equations

$$y = y_1 \qquad (3\text{-}4)$$

$$x = x_1 + s \cos \alpha \qquad (3\text{-}5)$$

$$z = z_1 + s \sin \alpha \qquad (3\text{-}6)$$

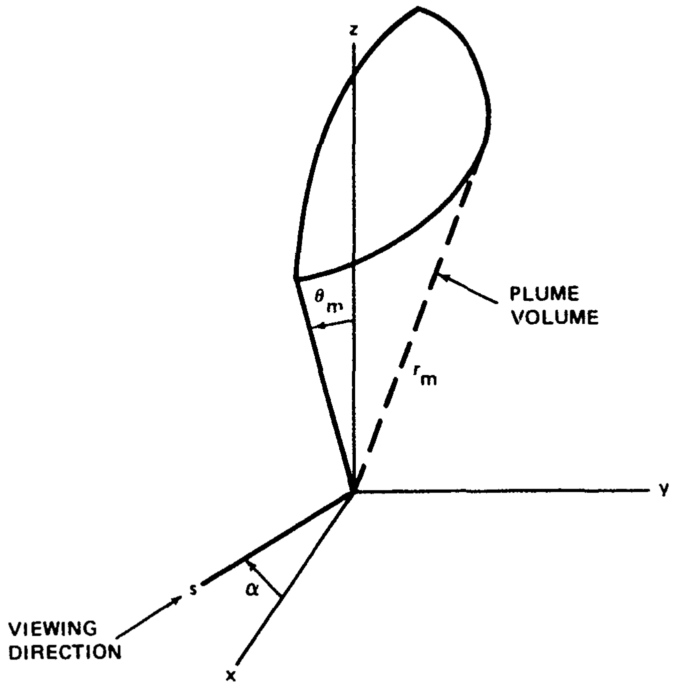

Figure 3-1. Plume Orientation and Viewing Direction

These equations can be switched to spherical coordinates using the standard relations

$$x = r\sin\theta\cos\phi \tag{3-7}$$

$$y = r\sin\theta\sin\phi \tag{3-8}$$

$$z = r\cos\theta \tag{3-9}$$

The following discussion derives the spherical and cartesian forms for ds along straight lines of sight:

3.1.1 Spherical Coordinates

From Equation 3-8 $$\frac{y_1{}^2}{(r\sin\theta)^2} = \sin^2\phi; \tag{3-10}$$

From Equations 3-5, 3-6, 3-7, and 3-9

$$\frac{\left[\cot\alpha\,(r\cos\theta - z_1) + x_1\right]^2}{(r\sin\theta)^2} = \cos^2\phi \tag{3-11}$$

Adding Equations 3-10 and 3-11

$$y_1^2 + \left[\cot\alpha\,(r\cos\theta - z_1) + x_1\right]^2 = (r\sin\theta)^2 = r^2 - (r\cos\theta)^2 \qquad (3\text{-}12)$$

let

$$\cot\alpha \;\; r\cos\theta = F$$

$$-\cot\alpha \;\; z_1 + x_1 = A$$

$$-y_1^2 + r^2 = B$$

Equation 3-12 becomes

$$F^2 + 2AF + A^2 - B + \frac{F^2}{\cot^2\alpha} = 0$$

or

$$F^2(1+\tan^2\alpha) + 2AF + A^2 - B = 0; \; F^2/\cos^2\alpha + 2AF + A^2 - B = 0$$

Therefore

$$F = \frac{-2A \pm \sqrt{4A^2 - 4A^2\left(\frac{1}{\cos^2\alpha}\right) + \frac{4B}{\cos^2\alpha}}}{1/\cos^2} =$$

$$-2A\cos^2\alpha \;\pm 2\cos\alpha\sqrt{B - A^2\sin^2\alpha}$$

Therefore

$$\sin\alpha\; r\cos\theta = -2A\cos\alpha \pm 2\sqrt{B - A^2\sin^2\alpha}$$

or

$$\cos\theta = -\frac{2A\cot\alpha}{r} \pm \frac{2}{r}\sqrt{\frac{B}{\sin^2\alpha} - A^2} \equiv u(r)$$

$$\theta = \cos^{-1}\left(-\frac{2A\cot\alpha}{r} \pm \frac{2}{r}\sqrt{\frac{B}{\sin^2\alpha} - A^2}\right)$$

From consideration of the case $\alpha = 90$ degrees and noting that $\cos\theta > 0$, only + root is valid.

$$\frac{d\theta}{dr} = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dr}$$

or

$$\frac{d\theta}{dr} = \frac{-1}{\sqrt{1-u^2}} \left[-\frac{u}{r} + \frac{1}{r}\left(\frac{B}{\sin^2\alpha} - A^2\right)^{-1/2} \frac{2r}{\sin^2\alpha} \right] \tag{3-13}$$

also

$$\sin\phi = \frac{y_1}{r\sin\theta} = \frac{y_1}{r\sqrt{(1-u^2)}} \equiv v(r)$$

$$\frac{d\phi}{dr} = \frac{1}{\sqrt{1-v^2}} \frac{dv}{dr}$$

where

$$\frac{dv}{dr} = \frac{-y_1}{r^2\sqrt{1-u^2}} + \frac{y_1 u}{r(1-u^2)^{3/2}} \frac{du}{dr}$$

From Equation 3-13

$$\frac{d\theta}{dr} = \frac{uw}{r(1-u^2)^{1/2}}$$

where

$$w(r) = \left[1 - \frac{2r}{u\sin^2\alpha \sqrt{B/\sin^2\alpha - A^2}}\right]$$

then

$$\frac{dv}{dr} = \frac{-y_1}{r^2\sqrt{1-u^2}} - \frac{y_1 u^2 w}{r^2(1-u^2)^{3/2}}$$

$$\frac{du}{dr} = -\frac{uw}{r}$$

or finally

$$ds = dr \left[1 + r^2 \frac{1}{1-v^2} \frac{y_1^{\,2}}{r^4(1-u^2)} \left(1 + \frac{u^2 w}{1-u^2}\right)^2 + r^2 \frac{y_1^{\,2}}{r^2(1-u^2)} \frac{u^2}{r^2} w^2 \right]^{1/2} \qquad (3\text{-}14)$$

From Equations 3-3, 3-4, 3-5, and 3-6 the limits of integration, r, and r_2 can be found.

We have

$$x^2 + y^2 + z^2 = r^2 \qquad (3\text{-}15)$$

$$x^2 + y^2 = z^2 \tan^2\theta_m, = r^2 - z^2$$

$$r^2 = z^2 (1 + \tan^2\theta_m)$$

Therefore

$$z = r\cos\theta_m$$

Also

$$y = y_1$$

and

$$\frac{x-x_1}{\cos\alpha} = \frac{z-z_1}{\sin\alpha}$$

or

$$x = \cot\alpha \, (z-z_1) + x_1 \qquad (3\text{-}16)$$

Substituting x, y, and z into equation of the cone, Equation 3-3 we have a quadratic equation for r, namely

$$(\cot^2\alpha \cos^2\theta_1 + \cos^2\theta_1 - 1)r^2 + (2x_1 \cot\alpha\cos\theta_1 - 2\cot^2\alpha\, z_1 \cos\theta_1)r$$

$$+ (\cot^2\alpha\, z^2_1 - 2x_1\cot\alpha z_1 + x_1^2 + y_1^2) = 0 \qquad (3\text{-}17)$$

The solution gives r_1 and r_2, as function of z_1, x_1, and y_1

3.1.2 Cartesian Coordinates

From Equation 3-16

$$z = \tan\alpha (x - x_1) + z_1 \qquad (3\text{-}18)$$

or

$$\frac{dz}{dx} = \tan\alpha$$

Therefore

$$ds = dx \sqrt{1 + \tan^2\alpha} = \frac{dx}{\cos\alpha} \qquad (3\text{-}19)$$

or

$$ds = \frac{dz}{\sin\alpha} \qquad (3\text{-}20)$$

We obtain r and θ from Equation 3-18

$$r^2 = z^2 + y_1^2 + \left[\cot\alpha (z - z_1) + x_1\right]^2 \qquad (3\text{-}21)$$

$$\cos\theta = z/r \qquad (3\text{-}22)$$

It is apparent that it is easier to use cartesian coordinates, and Equations 3-19 or 3-20 together with Equations 3-18, 3-21, and 3-22 to integrate through the plume, than to use the single, complex Equation 3-14.

To avoid singularitics at $\alpha = 0$ degree or 90 degrees, x will be used as the independent variable for $0 \le \alpha \le 45$ degrees and z for the independent variable for 45 degrees $< \alpha \le 90$ degrees. Since the plume is transparent, negative values of α are not needed, the integrals being equal to values for 90 degrees $+ \alpha$.

For cartesian coordinates, the intercept equations are more complex than for spherical coordinates; substituting

$$y = y_1 \tag{3-23}$$

$$x = \cot\alpha\,(z - z_1) + x_1 \tag{3-24}$$

into equation of spherical cap

$$x^2 + y^2 + z^2 = r_m^2 \tag{3-25}$$

and conical sides

$$x^2 + y^2 = z^2 \tan^2\theta_m \tag{3-26}$$

yields the following equations for the resulting four intercept points of a straight line of sight:

For $\underline{0 < \alpha < 45}$ degrees (choose $x_1 = 0$)

$$(1 + \tan^2\alpha)x^2 + (2z_1 \tan\alpha)\,x + y_1^2 + z_1^2 - r_m^2 = 0 \tag{3-27}$$

$$(\cot^2\theta_m - \tan^2\alpha)x^2 - (2z_1 \tan\alpha)\,x + y_1^2 \cot^2\theta_m - z_1^2 = 0 \tag{3-28}$$

and for $\underline{45 \text{ degrees} < \alpha \le 90 \text{ degrees}}$ (choose $z_1 = 0$)

$$(1 - \cot^2\alpha)z^2 + (2x_1 \cot\alpha)z + y_1^2 + x_1^2 - r_m^2 = 0 \tag{3-29}$$

$$(\tan^2\theta_m - \cot^2\alpha)z^2 - (2x_1 \cot\alpha)z - \left(y_1^2 + x_1^2\right) = 0 \tag{3-30}$$

For each range of α, solution of the two quadratic equations will yield four values for x or z. The correct limits of integration are found by selecting those pairs of x or z for which the corresponding values of r and θ (by Equations 3-18, 3-21, and 3-22) are on the real plume boundary, namely

$$r_m \text{ and } \theta \leq \theta_m$$

or

$$\theta_m \text{ and } r \leq r_m$$

3.1.3 Integration Schemes and Step Sizes

Most of the integrals of the form

$$I = \int_{x_1}^{x_2} f(x)\,dx \tag{3-31}$$

are evaluated using Gaussian eight-point quadrature. Defining

$$y = (2x - x_1 - x_2)/(x_2 - x_1) \tag{3-32}$$

we have

$$I = \frac{x_2 - x_1}{2} \int_{-1}^{1} f(x)\,dy \tag{3-33}$$

Then

$$I \approx \frac{x_2 - x_1}{2} \sum_{i=1}^{8} w_i\, f(x_i) \tag{3-34}$$

where

$$y_1 = y_8 = 0.18343\ 46424\ 95650$$

$$y_2 = y_7 = 0.52553\ 24099\ 16329$$

$$y_3 = y_6 = 0.79666\ 64774\ 13627$$

$$y_4 = y_5 = 0.96028\ 98564\ 97536$$

and

$$w_1 = w_8 = 0.36268\ 37833\ 78362$$

$$w_2 = w_7 = 0.31370\ 66458\ 77887$$

$$w_3 = w_6 = 0.22238\ 10344\ 53374$$

$$w_4 = w_5 = 0.10122\ 85362\ 90376$$

The values of x_i are related to y_i through Equation 3-32.

For integrating through the plume, Simpson's rule is used

$$\int_{x}^{x+2\Delta x} f(y)dy = (f(x) + 4f(x+\Delta x) + f(x+2\Delta x))\ \frac{2\Delta x}{3} \qquad (3\text{-}35)$$

The step size, Δx, is varied after each successive integration from x to x + $2\Delta x$. A geometric variation of Δx assures that the smallest step size occurs as the integration along the line of sight passes through the plume center (x = 0 or z = 0 planes).

Let the total maximum distance along the line of sight be x_T. Starting at the middle of the plume and integrating out, a distance $x_T/2$ is covered in about n/2 steps (n is a program input). If each Δx step is r times bigger than the previous, the rule for geometric progression gives

$$\frac{x_T}{2} = \Delta x_{min} \frac{r^{n/2}-1}{r-1} \qquad (3\text{-}36)$$

where Δx_{min} is the initial, smallest step size. Choosing a 10-percent increase in Δx each step, r = 1.1

or

$$\Delta x_{min} = \frac{x_T}{20(1.1^{n/2}-1)} \qquad (3\text{-}37)$$

Starting at position $x = -|x_o|$, a large step size is used which decreases until $x = 0$ and then increases again. The initial value of Δx is found from the formula for the sum in terms of the last term (Δx_{in})

$$x_o = \frac{\Delta x_{in} r - \Delta x_{min}}{r - 1} \tag{3-38}$$

or

$$\Delta x_{in} = (0.1\, x_c + \Delta x_{min})/1.1 \tag{3-39}$$

A similar stepping scheme is used to obtain a higher density of lines of sight near the rocket exit plane or plume centerline than used in the rarified plume.

3.1.4 Relative Velocity

The rocket-body velocity must be specified in the coordinate system of Figure 3-1 so that relative velocity of plume gases and atmosphere can be calculated. Figure 3-2 shows the position of the vehicle velocity vector V_R, and the definition of straight up, $\hat{u}$.

Integration is performed only over a plume where $y > 0$. To obtain other half of plume, ψ_R and ψ_u should be increased 180°.

Relative velocity is just the vector sum of $V_p\hat{r}$ and $\vec{V_R}$ where $\hat{r}$ is the unit vector along the polar coordinate r to a point in the plume being considered. Altitude at this same point is missile altitude h_0 plus projection of $\vec{r}$ onto $\hat{u}$.

$$h = h_o + \vec{r} \cdot \hat{u} \tag{3-40}$$

3.2 FLOW CHARTS

This section presents the logic flow of the FLAME code. Input data are listed in Tables 3-1 through 3-4. The Figure 3-3 is a simplified diagram of the overall code. The charts (Figures 3-4 through 3-9) expand upon certain regions of the computer.

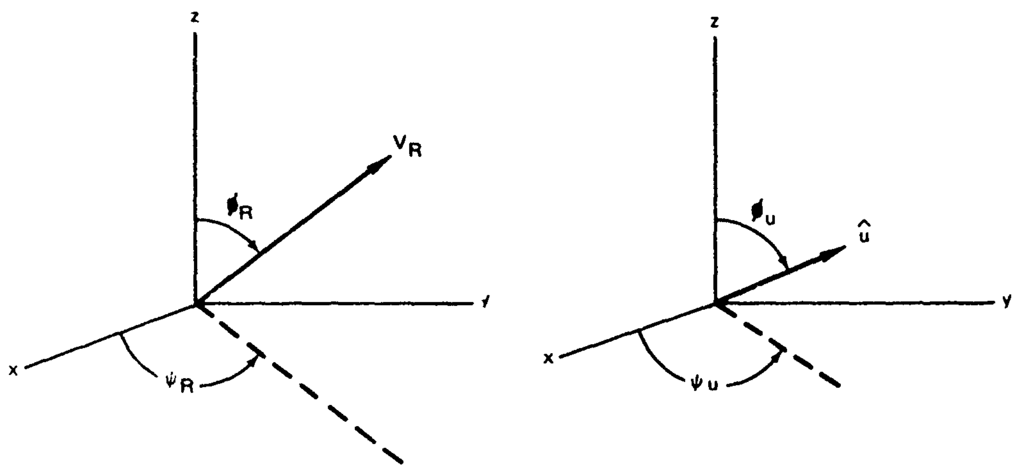

Figure 3-2. Location of Vehicle Velocity Vector and Upward Direction in Coordinate System of Figure 3-1.

Table 3-1

LISTING OF INPUT DATA

Read in		FORTRAN	Real or Integer
t,	time after ignition (sec)	TIG	R
r_e,	nozzle exit radius (cm)	RE	R
γ,	ratio of specific heats (> 1)	GAMMA	R
M_e,	exit mach number	ME	R
θ_e,	nozzle lip angle (deg)	LIP	R
$d_e(t)$,	exit plane density as a table versus time after ignition (g/cm^3 vs sec)	PDE TID	R R
I_{ND},	night/day flag (0 or 1)	IND	I
I_{SL},	solid/liquid flag 0 or -2 (gas); 1 or -1 (solid)	ISL	I
λ_{min}, λ_{max},	band limits (μm)	WLMIN WLMAX	R R
n,	approximate integration steps through layer	INT	I
m,	approximate number of lines of sight used in one direction	LINES	I
T_c	chamber temperature (°K)	TC	R
x_m,	radiating gaseous species mole fraction*	XM	R
T_E,	Earth effective temperature (°K)	TE	R
α,	look angle ($0 \leq \alpha \leq 90$ degrees)	AL	R
h_0,	altitude of engine (ft) (from 0.3 Mft to 3 Mft)	HO	R
V_R,	rocket velocity (fps)	VR	R
ϕ_u	polar angle of vertical direction (deg)	PHIU	R
ψ_u	azimuthal angle of vertical direction (deg)	XIU	R
ϕ_R,	polar angle of rocket-body velocity (deg)	PHIR	R
ψ_R,	azimuthal angle of rocket-body velocity (deg)	XIR	R
END,	flag signifying end of run (0 or 1)	END	I

*FLAME assumes an average molecular weight (WMOL) of 17 for the plume species (approximately correct for pure gas plumes, or the gaseous constituents of solid-propellant plumes). When gaseous radiation from a solid propellant plume is calculated ($|I_{SL}| = 1$ and $x_m \neq 0$), the molecular weight should be corrected to include solids. This is done by modifying x_m, since only the ratio XM/WMOL is used by FLAME. For a typical solid-propellant plume, WMOL = 26, so XM should be reduced by the ratio (17/26).

Table 3-2

PROGRAM REGION 1. INITIAL CALCULATIONS—PARTICLES

The following data will be calculated for each particle radius, p_i, corresponding to the i^{th} particle size:

EPT,	table of total emissivity versus temperature
TCOOL,	table of particle temperature versus time after solidification
BPASS,	black body intensity in band of interest
ETL	table of average emissivity in region from λ_{min} to λ_{max} versus temperature
TMIN,	equilibrium temperature of particle
DELT,	time required to solidify droplet

Table 3-3

PROGRAM REGION 2. INITIAL CALCULATIONS—GAS

Calculate limiting plume-gas velocity, VP.
For the band width specified, calculate

ET,	table of total available rotational energy as function of temperature
JM,	table of radiative rate (radiance per molecule) as function of temperature
JT,	table of radiative rate as a function of time

Table 3-4

PROGRAM REGION 3. SELECT x_1 OR z_1 for $\alpha \geq 90 - \theta_m$

Test IA to determine magnitude of α. For IA = 0, define z_1. For IA = 1, define x_1. Both x_1 and z_1 are labeled X1 in the program.

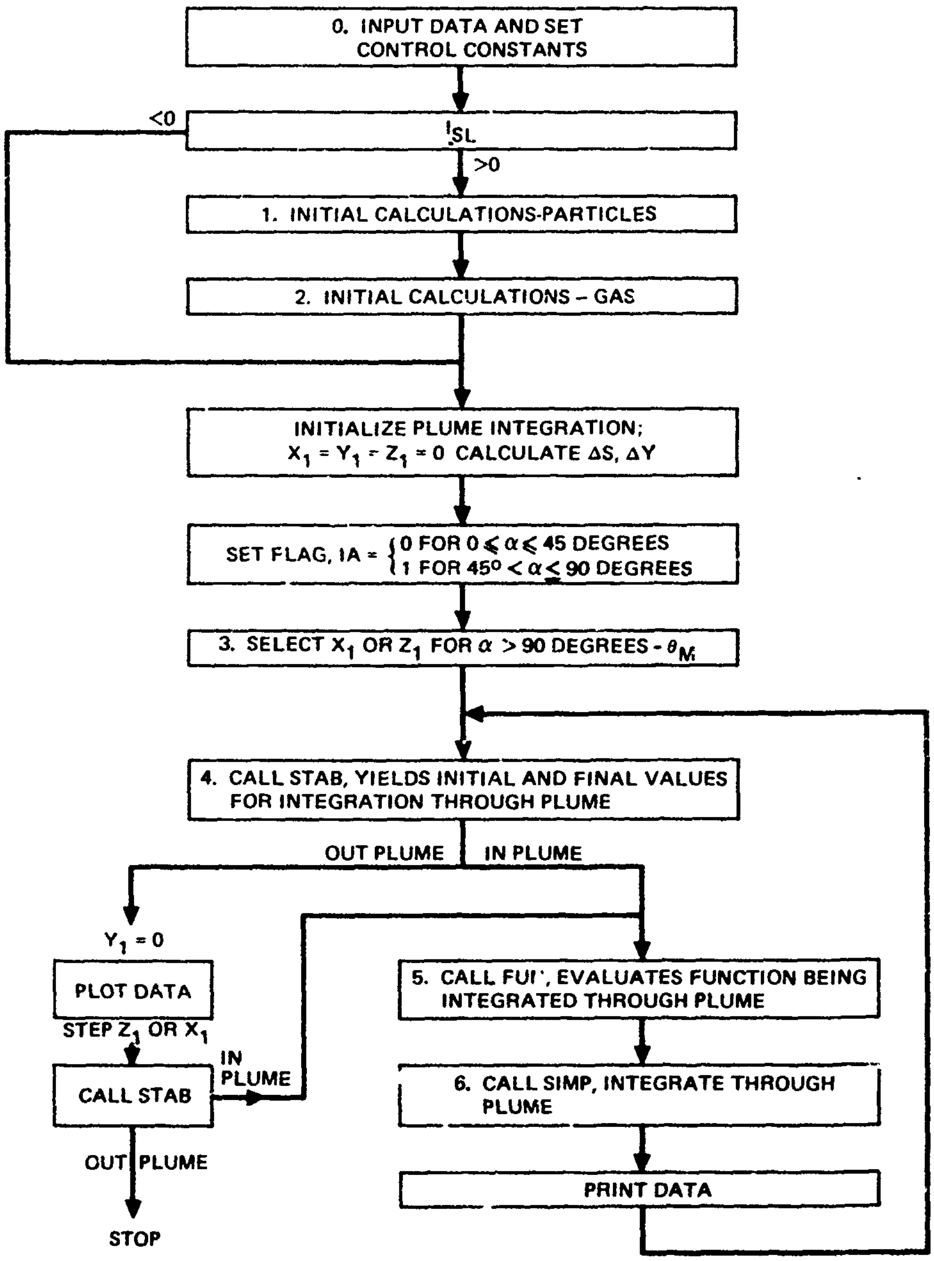

Figure 3-3. Master Flow for Program P2170, FLAME

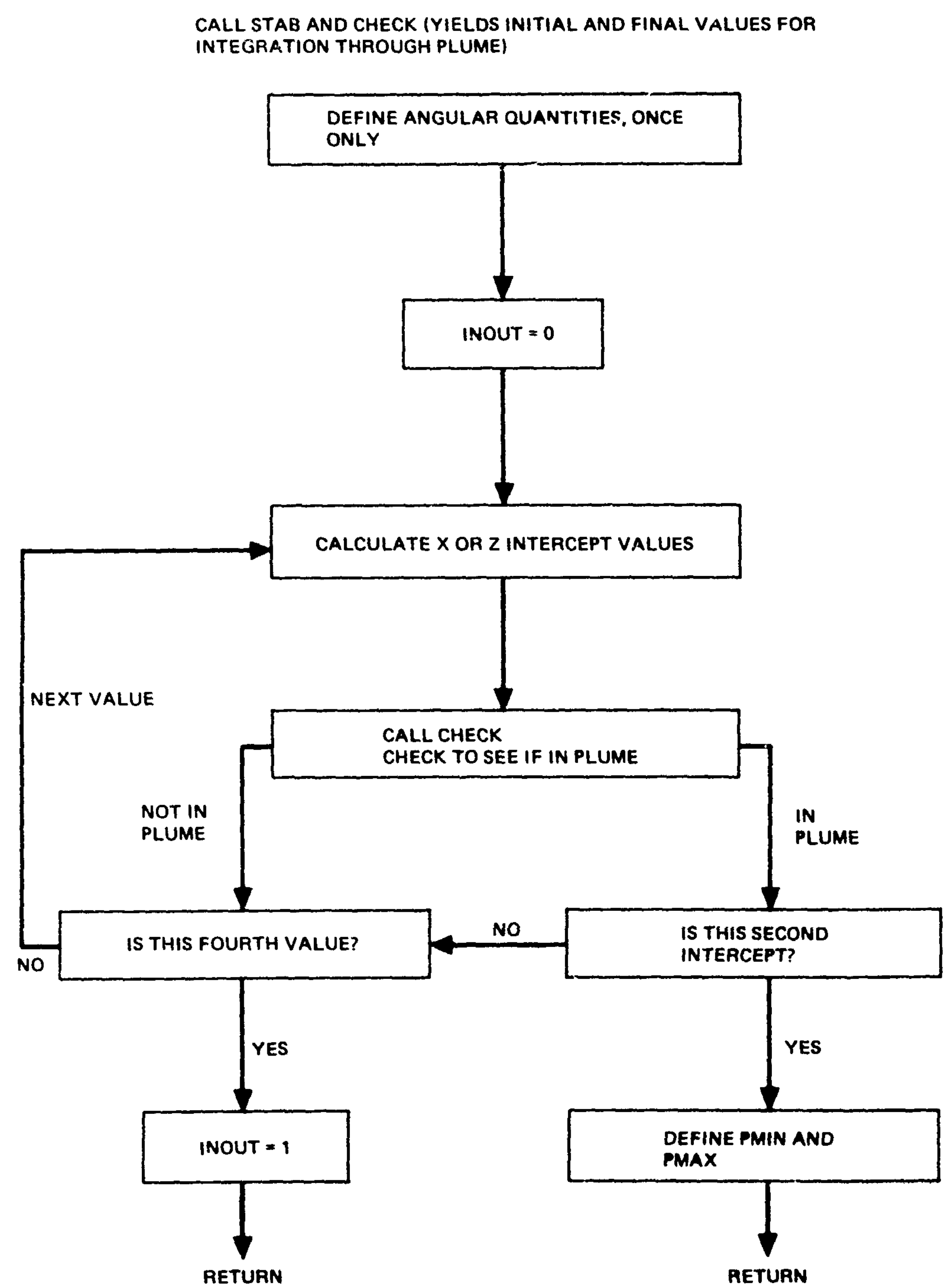

Figure 3-4. Program Region 4 Flow

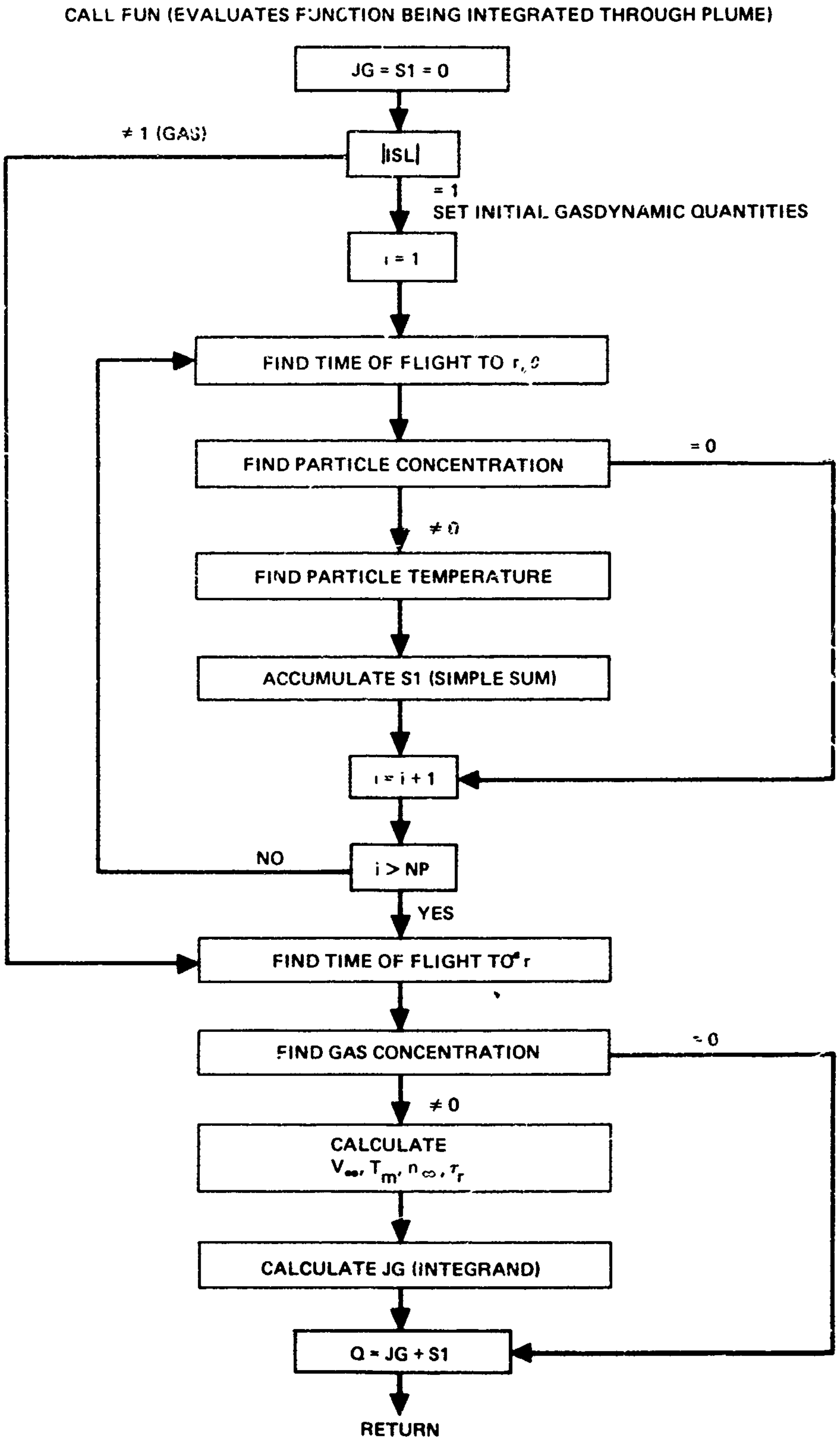

Figure 3-5. Program Region 5 Flow

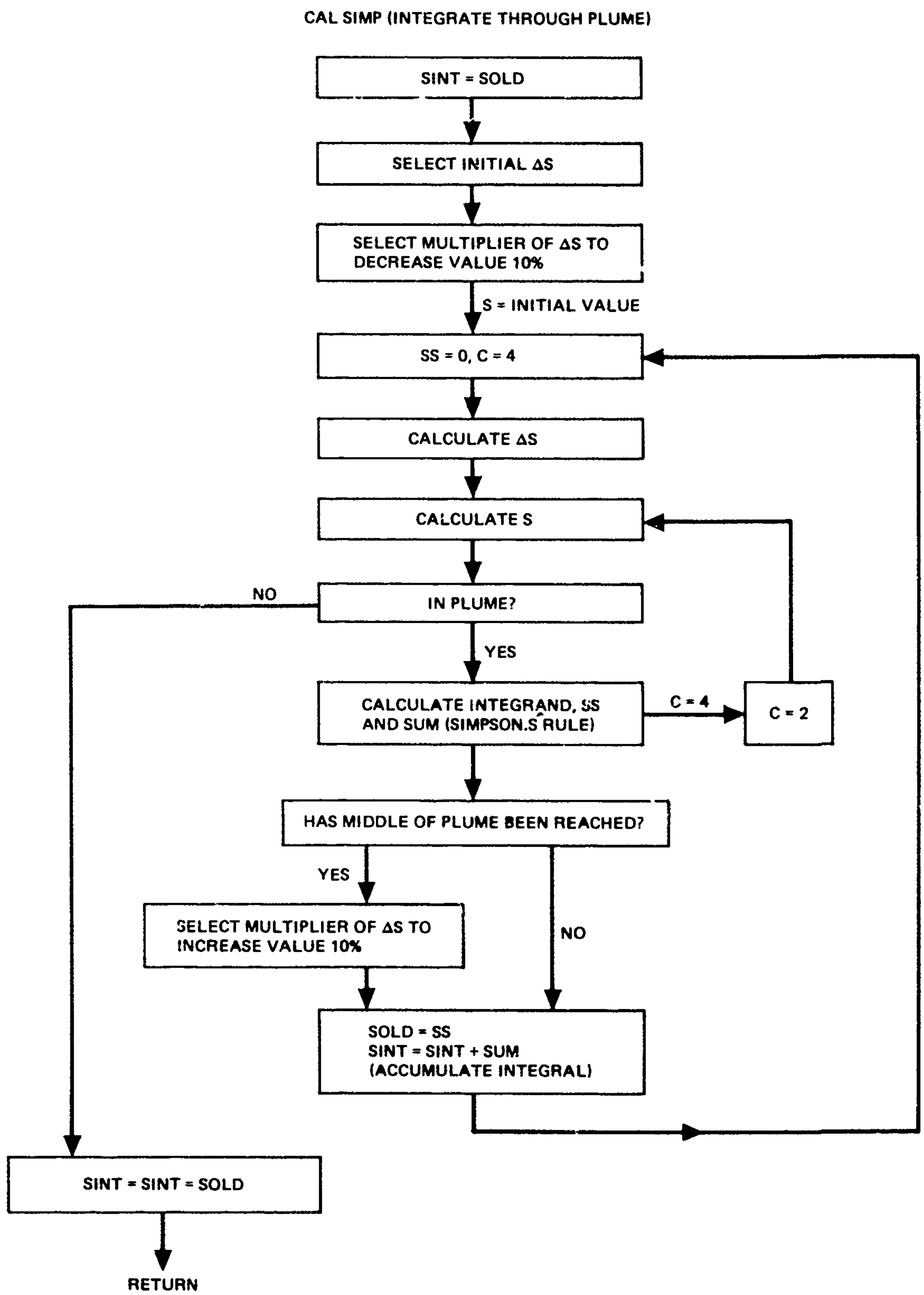

Figure 3-6. Program Region 6 Flow

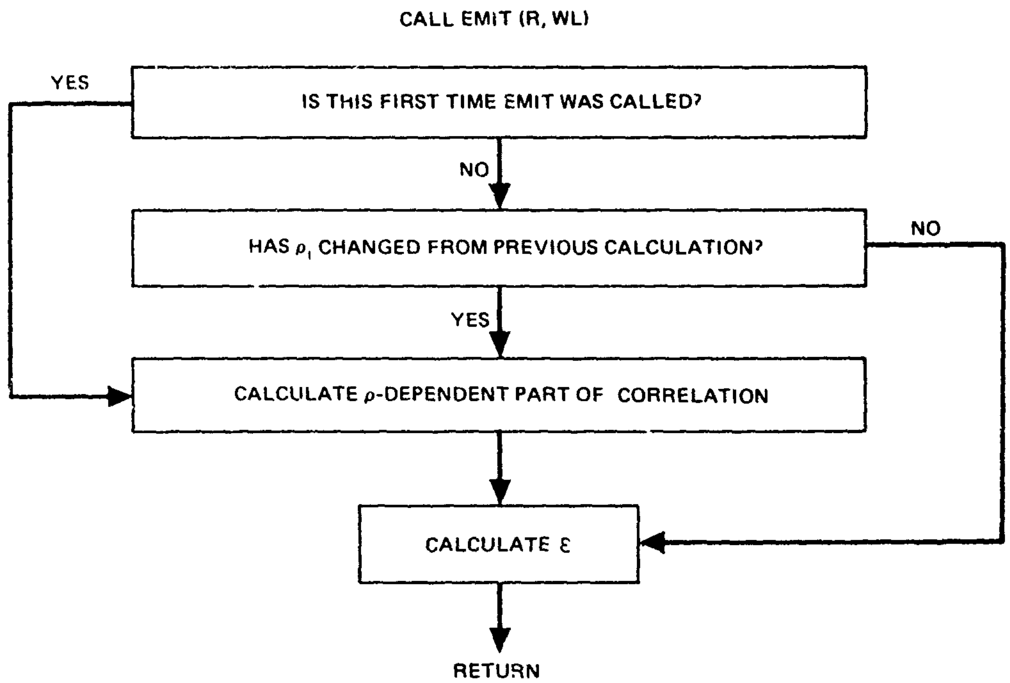

Figure 3-7. Particle Emissivity Routine Flow Chart

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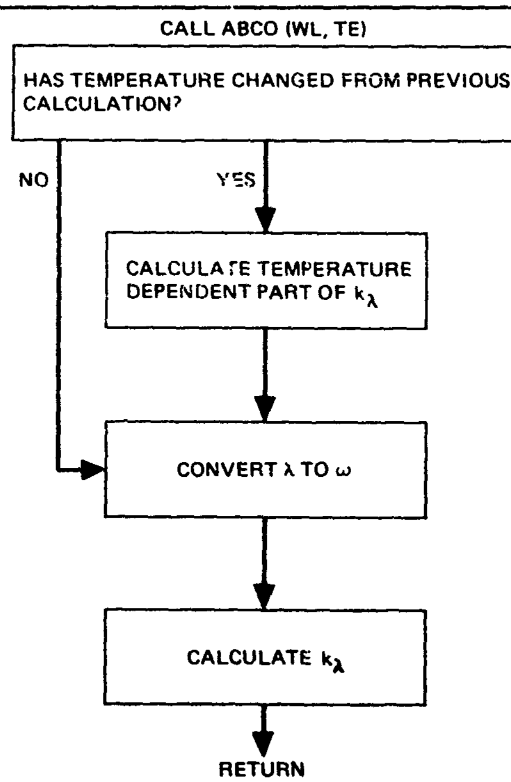

Figure 3-8. Absorption Coefficient Routine Flow Chart

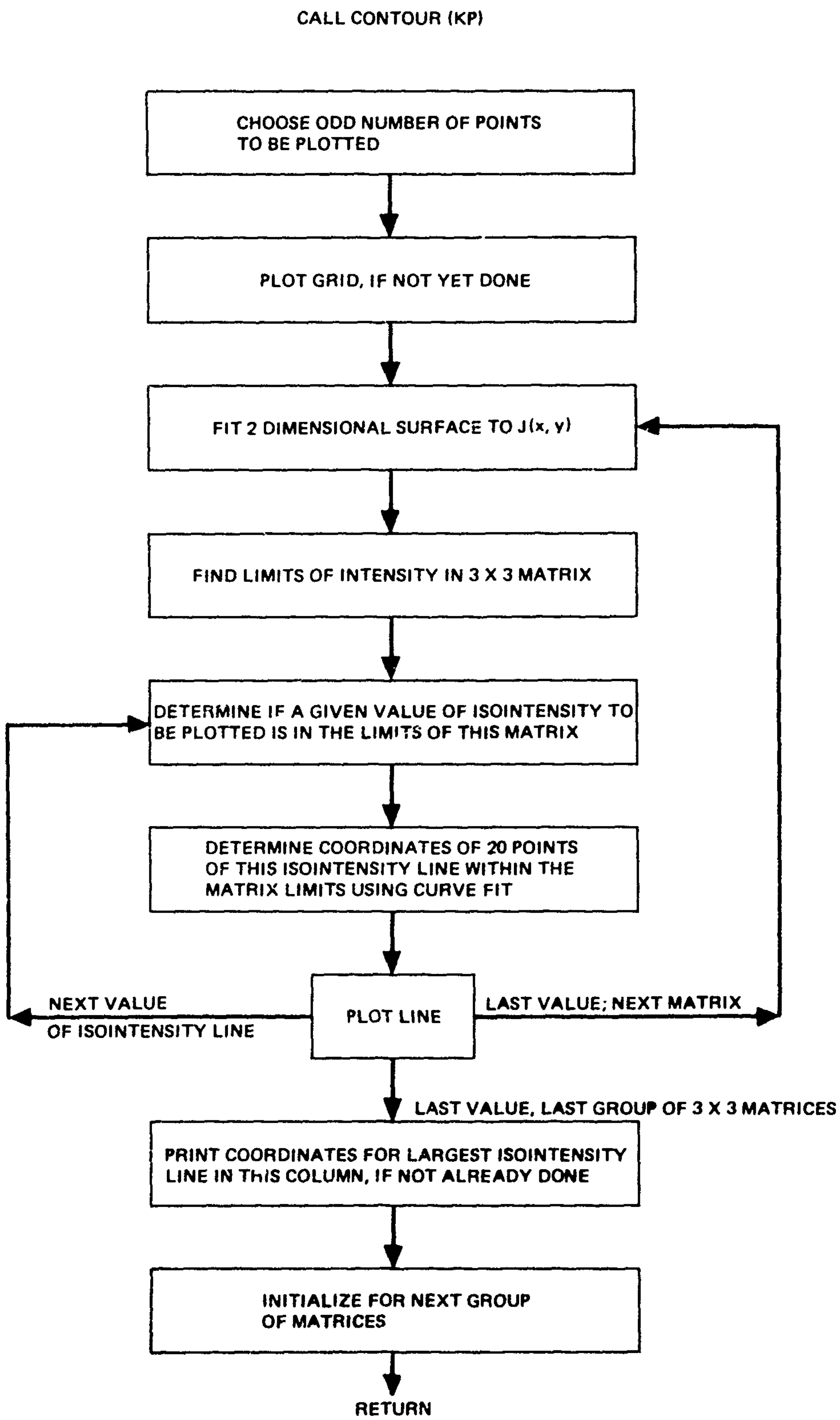

Figure 3-9. Contour Plotter Flow Chart

Section 4

EQUATIONS TO BE SOLVED

The following subsections relate to the program regions shown on the master flow chart, Figure 3-3.

4.1 PROGRAM SECTION 1. INITIAL CALCULATIONS—PARTICLES

4.1.1 EPS, Particle Emissivity Versus Temperature

$$\epsilon_i (T) = \int_0^\infty \epsilon_{i\lambda} R_\lambda^o (T) \, d\lambda / \sigma T^4 \tag{4-1}$$

where

$$R_\lambda^o (T) = \frac{2\pi c^2 h}{\lambda^5} \left[e^{\frac{hc}{\lambda kt}} - 1 \right]^{-1} = \frac{c_1}{\lambda^5} \left[e^{\frac{c_2}{\lambda T}} - 1 \right]^{-1} \tag{4-2}$$

c = velocity of light

σ = Stefan-Boltzmann constant

= 5.672×10^{-12} j/cm^2 ^{o}K^4 sec

$\epsilon_{i\lambda}$ = correlation eq (2-6)

c_1 = 3.742×10^4 w $(\mu m)^4/cm^2$

c_2 = 14390 μm ^{o}K

See Subsection 3.1.3 for description of Gaussian Integration.

4.1.2 TCOOL, Particle Temperature Versus Time After Solidification

$$\Delta t_i (T) = \frac{c_p \rho_i}{3\sigma} \int_{T_f}^{T} \frac{dT}{\epsilon_i T^4} \tag{4-3}$$

where

c_p = particle specific heat

T_f = fusion temperature

4.1.3 BPASS Black Body Intensity in Band of Interest

$$B_{\Delta\lambda}^{o}(T) = \frac{1}{\pi}\int_{\lambda_{min}}^{\lambda_{max}} R_{\lambda}^{o}(T)\, d\lambda \qquad (4\text{-}4)$$

4.1.4 EPSI, Average Particle Emissivity

$$\epsilon_{i,\Delta\lambda}(T) = \frac{1}{\pi}\int_{\lambda_{min}}^{\lambda_{max}} \epsilon_{i\lambda}\, R_{\lambda}^{o}(T)\, d\lambda / B_{\Delta\lambda}^{o}(T) \qquad (4\text{-}5)$$

4.1.5 TMIN, Particle Equilibrium Temperature

at day,

$$\epsilon_i\, T_{min,i}^{4} = \frac{a_i \Gamma}{2\sigma} + \frac{a_i'}{2}\; T_E^{4} \equiv K \qquad (4\text{-}6)$$

where

a_i = solar absorptivity $[\sim \epsilon_i\,(6000^o)]$

a_i' absorptivity to Earthshine $[\sim \epsilon_i\,(T_E)]$

Γ = solar constant = 0.1353 w/cm^2

T_E = effective Earth temperature

at night, Equation 4-6 is used with

$\Gamma = 0$

Newton-Rapheson solution is of form

$$T_j = \frac{1}{3}\left[2T_{j-1} + \frac{K}{T_{j-1}^{3}\,\epsilon_i\,(T_{j-1})}\right] \qquad (4\text{-}7)$$

4.1.6 DELT, Time to Solidify

$$\Delta t_{i,s} = \frac{\rho_i m \lambda}{3\epsilon_i\, \sigma\, T_f^{4}} \qquad (4\text{-}8)$$

where

m = particle density

λ = particle heat of fusion

4.2 PROGRAM SECTION 2. INITIAL CALCULATIONS--GAS

4.2.1 Plume Gas Limiting Velocity

$$V_p = a_o \left(\frac{2}{\gamma - 1}\right)^{1/2} \tag{4-9}$$

where

$$a_o = \sqrt{\gamma R T_c} \tag{4-10}$$

4.2.2 ET, Rotational Energy Versus Temperature

$$E_T = \frac{hcB}{2\sigma^{3/2}} \left[x_1^{1/2} \ e^{-x_1} - x_2^{1/2} \ e^{-x_2} + \frac{\sqrt{\pi}}{2} \left(\text{erf} \sqrt{x_2} - \text{erf} \sqrt{x_1}\right)\right] \tag{4-11}$$

where

B = molecular rotational constant

$$\sigma = hcB/kT \tag{4-12}$$

$$x_{1,2} = J_{1,2}^{\ 2} \sigma \tag{4-13}$$

$$J_{1,2} = \frac{1}{2B\lambda_{min,\ max}} \tag{4-14}$$

$$\frac{hc}{2} = 0.9928 \text{ w sec cm}$$

$$\frac{hc}{k} = 1.4387 \text{ cm } ^{o}\text{K}$$

4.2.3 JM, Radiative Rate in Band

$$j(T) = \frac{1}{\pi} \int_{\lambda_{min}}^{\lambda_{max}} k_\lambda (T) \, R^o_\lambda (T) \, d_\lambda \tag{4-15}$$

where $k_\lambda(T)$ = the spectral linear absorption coefficient per molecule from Equation 2-8, multiplied by kT/p to change from k_λ per atm.

4.2.4 JT, Radiative Rate Versus Time

$$-\int_{T_{max}}^{T} \frac{dE_T}{4\pi j} = t \tag{4-16}$$

where T is found for value of E_T from results of Subsection 4.2.2 above, and then j is formed from results of Subsection 4.2.3.

4.2.5 Closed Form for Radiance

A least-square fit is made to the resulting data of Subsection 4.2.4 of the form

$$j(t) = A\,(\tau_R - t) \tag{4-17}$$

Substituting Equation 4-17 into 4-16 yields

$$\tau_r^{\,2} = \frac{E_T}{2\pi A} \tag{4-18}$$

which relates τ_r to T_m, the maximum temperature excited by atmospheric collision.

4.3 PROGRAM SECTION 3. SELECTION OF x OR z_1

These are the initial values of x_1 or z_1 to be used to define lines of sight just grazing plume.

For $\alpha \le 45$ degrees, we have the following:

$\theta_m < 90$ and $90 - \theta_m > \alpha$

$$z_1 = 0 \tag{4-19a}$$

$\theta_m > 90$ and $\theta_m - 90 < 90 - \alpha$

$$z_1 = -r_m/\cos\alpha \tag{4-19b}$$

Otherwise

$$z_1 = r_m \cos(\alpha + \theta_m)/\cos\alpha \tag{4-19c}$$

For $\alpha > 45$ degrees, we have the following:

$\theta_m < 90$ degrees and $\alpha > \theta_m$

$$x_1 = -r_m \cos(\alpha - \theta_m)/\sin\alpha \tag{4-20a}$$

Otherwise

$$x_1 = -r_m/\sin\alpha \tag{4-20b}$$

In addition, if there is no gaseous radiation (XM = 0), the plume angular extent is taken equal to the flow angle of the limiting streamline for the smallest particle (33.52 degrees).

4.4 PROGRAM SECTION 4. CALL STAB--INTERCEPT VALUES

For $0 \le \alpha < 45$ degrees

$$(1 + \tan^2 \alpha)\, x^2 + (2 z_1 \tan \alpha)\, x + y_1^2 + z_1^2 - r_m^2 = 0 \qquad \text{(3-27, Section 3)}$$

$$(\cot^2 \theta_m - \tan^2 \alpha)\, x^2 - (2 z_1 \tan \alpha)\, x + y_1^2 \cot^2 \theta_m - z_1^2 = 0 \qquad \text{(3-28, Section 3)}$$

and for 45 degrees $\le \alpha \le$ 90 degrees

$$(1 - \cot^2 \alpha)\, z^2 + (2 x_1 \cot \alpha)\, z + y_1^2 + x_1^2 - r_m^2 = 0 \qquad \text{(3-29, Section 3)}$$

$$(\tan^2 \theta_m - \cot^2 \alpha)\, z^2 - (2 x_1 \cot \alpha)\, z - (y_1^2 + x_1^2) = 0 \qquad \text{(3-30, Section 3)}$$

4.5 PROGRAM SECTION 5. CALL FUN--RADIANCE CALCULATIONS

$$z = \tan \alpha\, (x - x_1) + z_1 \qquad \text{(3-18, Section 3)}$$

$$r = \sqrt{z^2 + y_1^2 + \left[\cot \alpha\, (z - z_1) + x_1\right]^2} \qquad \text{(4-21)}$$

$$\theta = \cos^{-1} (z/r) \qquad \text{(4-22)}$$

4.5.1 Solid Particles

4.5.1.1 Calculate Throat Radius

$$r_t = r_e / \sqrt{\epsilon}$$

where

$$\epsilon = \frac{1}{M_e} \left(\frac{\gamma + 1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \text{(4-23)}$$

4.5.1.2 Particle Limiting Velocity

$$V_{(\rho_i, \theta)} = [a_i + b_i \exp(-c_i\, r/r)] \cos^{n_i} \theta \tag{4-24}$$

for $0 \leq \theta \leq \theta_i$ (limiting particle streamline inclination) where the coefficients are given in Table 4-1.

4.5.1.3 Particle Time of Flight

$$t_f = r/V_{(\rho_i, \theta)} \tag{4-25}$$

4.5.1.4 Concentration

$$\text{Density ratio} = \frac{d_i}{d_o} = f(\rho_i, r, \theta) \equiv g_i \left(\frac{r_t}{r}\right)^{e_i} (\cos\theta)^{f_i} \tag{4-26}$$

for $0 \leq \theta \leq \theta_i$ and the coefficients are given in Table 4-2. Concentration is related to density by

$$N_o = d_o / \left(\frac{4}{3}\right) \pi \rho_i^3\, d'\,(XML),\quad d' = \text{alumina density}$$

$$= 4.0045\ g/cm^3$$

$$XML = \text{alumina mass fraction}$$

4.5.1.5 Particle Temperature

If $t_f < \Delta t_{i,s}$ (Equation 4-8),

$T = T_f$

If $t_f > \Delta t_{i,s}$,

T = maximum of

a. TCOOL $(t_f - \Delta t_{i,s})$, Equation 4-3

or b. TMIN, Equations 4-6 or 4-7

4.5.1.6 Evaluate Integrand

$$J_i = N_o (t' - t_f)\, f(\rho_i, r, \theta)\, \pi \rho_i^2\, \epsilon_{i,\Delta\lambda}(T)\, B^o_{\Delta\lambda}(T) \tag{4-27}$$

(See Equations 4-26, 4-4, and 4-5)

Table 4-1

PARTICLE LIMITING VELOCITY COEFFICIENTS

ρ_i(μm)	θ_i	a_i(cm/sec)	b_i(cm/sec)	c_i	n_i
0.690	33.52°	30.776×10^4	-4.6692×10^4	0.01874	-0.9048
1.330	28.22	29.701	-3.8094	0.01930	-1.3494
2.000	23.53	28.932	-3.2891	0.01976	-1.5419
2.660	19.76	28.248	-3.0367	0.02012	-1.4886
3.332	16.81	27.520	-2.9355	0.02036	-1.3175
4.000	14.75	26.764	-2.9337	0.02064	-1.0308
4.666	13.56	26.007	-2.9745	0.02092	-0.6425

Table 4-2

CONCENTRATION COEFFICIENTS

ρ_i(μm)	g_i	e_i	f_i
0.690	0.2201	2.33	2.84
1.330	0.4503	2.24	5.64
2.000	0.5395	2.17	6.75
2.660	1.0308	2.12	7.30
3.332	1.3339	2.08	6.95
4.000	0.8870	2.[illegible]	6.30
4.666	0.5396	2.04	-2.05

4.5.2 Excited Gases

4.5.2.1 Time of Flight

$$t_f = r/V_p, \text{ Equation 4-9} \tag{4-28}$$

4.5.2.2 Gas Concentration

$$n_p = n_{p,e} \frac{k}{2} \left(\frac{r_e}{r} \right)^2 [\cos (a\theta)]^{k-2} \tag{4-29}$$

where

$$k = \gamma(\gamma - 1) M_e^2 \tag{4-30}$$

$$a = \frac{90 \text{ degrees}}{\theta_e + (\nu_{max} - \nu_e)} \tag{4-31}$$

$$\nu_{max} = \left[\sqrt{\frac{\gamma + 1}{\gamma - 1}} - 1\right] 90 \text{ degrees} \tag{4-32}$$

$$\nu_e = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{\gamma - 1}{\gamma + 1} (M_e^2 - 1)} - \tan^{-1} \sqrt{M_e^2 - 1} \tag{4-33}$$

and $n_{p,e} = d_e N_o/\overline{M}$

N_o = Avogadro's number

$\overline{M}$ = mean molecular weight

and remaining parameters are input data

4.5.2.3 Relative Velocity

$$x = \cot \alpha (z - z_1) + x_1$$

$$V_\infty = \left[\left(\frac{xV_p}{r} + V1\right)^2 + \left(\frac{yV_p}{r} + V2\right)^2 + \left(\frac{zV_p}{r} + V3\right)^2\right]^{1/2} \tag{4-34}$$

where

$V1 = V_R \sin \phi_R \cos \psi_R$

$V2 = V_R \sin \phi_R \sin \psi_R$

$V3 = V_R \cos \phi_R$

4.5.2.4 Excitation Temperature

T_m = maximum of

A. T of ambient atmosphere (See Equation 4-40)

B. $T_M = 9.61194 \times 10^{-9} V_\infty^2 + 1.07636 \times 10^{-3} V_\infty$ (4-35)

(V_∞ in cm/sec)

4.5.2.5 Attenuation Factor

$$A_t = S \int_o^r n_\infty (h_s)\, ds \tag{2-7}$$

where the altitudes, h_s, are found from

$$h_s = h_o + s\left(\frac{h - h_o}{r}\right)$$

4.5.2.6 Radiative Relaxation Time

$$\tau_r = \tau_r (T_m), \text{ Equation 4-18} \tag{4-36}$$

4.5.2.7 Radiance Calculation

$$J_g = V_\infty\ n_\infty\ S\ \tau_r\ [1 - \exp (t_f / \tau_r)]\ n_p j\ \exp(-A_t)\ x_n \tag{4-37}$$

where for $t_f \geq \tau_r$

$$j = \frac{A\tau_r}{2}, \quad \text{(See Equation 4-17)} \tag{4-38}$$

and for $t_f < \tau_r$

$$j = A\left(\tau_r - \frac{t_f}{2}\right) \tag{4-39}$$

n_∞ is found from atmosphere table, $n_\infty(h)$

S = built-in value of cross section

$$h = h_o + x \sin \phi_u \cos \psi_u + y \sin \phi_u \sin \psi_u + z \cos \phi_u \tag{4-40}$$

(See Equation 3-40)

4.5.3 Total Radiance

If both particles and gas are included the total integrand is

$$J = J_p + J_g \tag{4-41}$$

4.6 PROGRAM SECTION 6. CALL SIMP--INTEGRATION THROUGH PLUME

Simpson's rule

$$\int_s^{s+\Delta s} f(x)\,dx = \frac{\Delta s}{3}\left[f(s) + 4\,f\left(s + \frac{\Delta s}{2}\right) + f(s + ds)\right] \tag{4-42}$$

Step size is controlled by a geometric progression, so that ea h step is 10 - larger or smaller than the previous, and minimum step size occurs at z = 0 or x = 0 planes.

4.7 CURVE FITTING ROUTINE, CF

An auxiliary routine was written to obtain a curve fit to the intensity data to facilitate plotting. The curve fit for intensity is of the form

$$I = \sum_{j=1}^{6} a_j\,\phi_j\,(x, y)$$

where

$$\phi_j = x^2,\ x,\ y^2,\ y,\ xy,\ 1$$

The least square fit condition is (for nine points)

$$\frac{d}{da_k}\sum_{i=1}^{9}\left[I_i - \sum_{j=1}^{6} a_j\,\phi_j\,(x_i,\ y_i)\right]^2 = 0$$

The resulting matrix of equations is

$$(C)\ (a) = (I)$$

where

$$\begin{pmatrix} x_i^4 & x_i^3 & x_i^2 y_i^2 & x_i^2 y_i & x_i^3 y_i & x_i^2 \\ & x_i^2 & x_i y_i^2 & x_i y_i & x_i^2 y_i & x_i \\ & & y_i^4 & y_i^3 & x_i y_i^3 & y_i^2 \\ & & & y_i^2 & x_i y_i^2 & y_i \\ & \text{(Symmetric)} & & & x_i^2 y_i^2 & x_i y_i \\ & & & & & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} I_i x_i^2 \\ I_i x_i \\ I_i y_i^2 \\ I_i y_i \\ I_i x_i y_i \\ I_i \end{pmatrix}$$

and each term in the C and I matrices is summed over all points, e.g.

$$C_{11} = \sum_{i=1}^{9} x_i^4$$

This technique of extrapolation is similar to that used by Dailey (Reference 12). Nine points (3 x 3 square) are selected and the curve fit made. Coordinates of the isointensity lines are then deduced from the curve fit by solving the quadratic equation for y, given I and x. These lines are plotted, and then the next group of nine points are selected. The procedure is repeated until the plume is covered.

Section 5
SAMPLE PROBLEM AND OPERATING INSTRUCTIONS

A list of all input quantities is given in Table 3-1* in Subsection 3.2, with their appropriate units. Additional description of the input parameters especially the Flags, is available in the problem output shown in Appendix B. The input is, in general, standard NAMELIST format (see listing of sample data in Appendix A). Following the NAMELIST input is a single card consisting of a case description (any 80 alphanumeric characters) which is printed at the start of the case. To signify the end of the run (no more cases), a NAMELIST-format input is required setting the flag, END, equal to 1. No case-description card should follow here.

A sample problem follows for an engine typical of a solid-propellant interceptor sustainer rocket. Since the radiating species mole fraction (XM) has been set equal to zero, only radiation from the alumina particles will show up. The specification of look angle equals zero implies a broadside look. The vertical angles equal to 90 degrees means that the plume axis is parallel to the Earth's surface. The values of 90 degrees for the velocity vector imply that the free-stream velocity is perpendicular to the plume axis. The final columns of printed output present x, y coordinates in cm followed by the intensity of w/cm^2 sr; many zero values of intensity result because the particles cannot expand through the total plume. The output from the sample case follows: (Note: The "error summary" in the output refers to reading in only five values for density versus time instead of maximum allowable value of 50 — no calculation errors result).

All intermediate calculations leading up to calculation of radiance (emissivities, etc.) need not be repeated on successive cases, if not changed. These calculations will be skipped if ISL is entered as -1 (solid propellant) or -2 (gas only)

*Note: If ISL = +1 or −1, the density table (PDE) should be for chamber density and not exit plane density.

Section 6
RECOMMENDATIONS FOR FUTURE WORK

This version of FLAME forms a useful framework for the calculation of plume radiance including a variety of mechanisms. The main accomplishments of this contract were to formulate the integration procedure and geometric-orientation choice to relieve the engineer of tedious extrapolations to attempt to relate flight data to calculations. Because of the large number of options included in FLAME, a comprehensive checkout was not possible during this contract (over 50 runs were made). A streamlining of the output and calculation procedures would be possible through a more extensive exercising of the FLAME options. A variety of values for look angle, altitude, times, orientations engine characteristics, relative velocity, etc. should be used as FLAME input to see if further refinements are needed.

Currently the code calculates only half of the plume in a single run which is sufficient for axisymmetry, such as for particle radiation only. The code should be refined to allow total plume extent to be calculated.

A better contour-plotting routine might be found by limiting the closeness of the isointensity lines, and constraining the curve fit to obtain smoother data. Automatic labeling of the contours would be a nice added feature.

The theoretical models used in FLAME are becoming rapidly antiquated. A molecular radiation model based on non-Boltzmann distributions should be investigated and incorporated.

Additional extensive flow-field revision would be useful based on sophisticated multiphase transient plume codes,

Section 7
REFERENCES

1. L. Roberts. The Action of a Hypersonic Jet on a Dust Layer. Institute of Aerospace Sciences, Paper No. IAS63-50, January 1963.

2. J. R. Kliegel and G. R. Nickerson. Axisymmetric Two-Phase Perfect Gas Performance Program. TRW Systems Report No. 02874-6006-R000, 3 April 1967.

3. J. R. Kliegel and G. R. Nickerson. User's Manual for the Two-Dimensional Two-Phase Plume Analysis Computer Program. Dynamic Science, December 1968.

4. H. H. Radke, et al. Exhaust Particle Size Data from Small and Large Solid Rocket Motors. Aerospace Corp. Report No. TOR-1001 (S2951-18)-3, July 1967.

5. J. A. F. Hill and J. S. Draper. Analytical Approximation for the Flow from a Nozzle into a Vacuum. J. Spacecraft and Rockets, 3, 1552-1554, October 1966.

6. G. N. Plass. Space System Division AF Systems Command, Report No. SSD-TDR-62-127. Vol. 6, 1962.

7. G. N. Plass. Applied Optics. Vol 3, p 867, 1964.

8. R. R. Brannon and R. I. Goldstein. Journal of Heat Transfer. Vol 92, P 257, 1970.

9. S. I. Morizumi and H. J. Carpenter. AIAA Preprint No. 64-61, 1964.

10. M. Thomas and R. L. Thommarson. Collisional Excitation of Molecular Radiation. McDonnell Douglas Astronautics Company Report No. MDC G2537, October 1971.

11. General Research Corporation. Minutes of Optical Signatures Program Meeting. Report WGRC 71-2446, October 13, 1971.

12. G. Dailey. A Computer Program to Plot Contour Maps of Stresses in Two-Dimensional Structures Analyzed by the Method of Finite Elements. Technical Memorandum TG 1128, The Johns Hopkins University, July 1970.

Appendix A
LISTING OF FLAME CODE

```
C PROGRAM FLAME, VERSION 1
      PROGRAM P2170(INPUT,OUTPUT,FILMPL,TAPE5=INPUT,TAPE6=OUTPUT,
     1              TAPE48=FILMPL,TAPE16)
      DIMENSION PX(3),PY(100),PL(3,100),FC(20),IP2(20)
      DIMENSION PDE(50),TID(50),         WLE(8),EPT(20,10),TET(20),TCOOL(
     120,10),TCI(20),BPASS(20),FTL(20,10),TMIN(10),DELT(10),ET(20),IR(20
     2),JM(20),JT(20),TJ(20),XP(8),W(8),RP(10),TCOO(20),TCIX(20,10),CASE
     3(10),TIDP(51),PDEP(51),T1(10),T2(10),WP(20),CU(2)
      EQUIVALENCE (TCIX,TIDP),(TCIX(1,4),PDEP),(T1,JT),(T2,JT(11))
      REAL IA,ME,JM,JT,LIP
      INTEGER END
      COMMON/MCG/ JQ,IPL,PX,PL,PY,FC,XL,XR,YB,YT,IP,IP2
      COMMON/MSTCF/IA,AL,RD,CONF,RM,X1,Y1,RA
      COMMON/MF/ISL,NP,G,RE,GAMMA,ME,AN,TIG,TID,PDE,NR,XML,DELT,TMIN,TCI
     1,TCOOL,PI,RP,TET,ETL,BPASS,TF,VP,P2,  XI2,TR,ET,A,V1,V2,V3,HO,WMOL
     2,XM,XP,W,GL
      DATA XP,W,RA,PI,RD,WP/-.9602898565,-.7966664774,-.5255324099,-.183
     14346425,.1834346425,.5255324099,.7966664774,.9602898565,.101228536
     23,.2223810344,.3137066458,.3626837834,.3626837834,.3137066458,.222
     33810344,.1012285363,90.,3.1415927,57.2957795131,.001,19*1./
      DATA NP,RP/7,46.604E-5,40.E-5,33.315E-5,26.6E-5,20.001E-5,13.3015E
     1-5,6.9007E-5/,B/9.49/
      DATA TR/100.,400.,800.,1200.,1600.,2000.,2400.,2700.,3000.,3300.,
     13600., 3900.,4200.,4500.,5000.,5500.,6000.,7000.,9000.,11000./
      DATA TET/99.,150.,200.,250.,300.,350.,400.,450.,500.,600.,700.,
     1800.,1000.,1200.,1400.,1700.,2000.,2500.,3200.,6001./,
     2 TF,DF,CP/2323.,4240.,3.2/
C ***** REGION 0,   INPUT DATA   *****
      NAMELIST /IN/TIG,RE,GAMMA,ME,LIP,TE,AL,HO,VR,PHIU,XIU,TC,PHIR,XIR,
     1WLMIN,WLMAX,XM,IND,ISL,INT,LINES,NR,TID,PDE,END,IPLOT
      GL = ALOG(10.)
      IPT = 0
    1 READ(5,IN)
      IF (END.EQ.1) STOP
      JP = 1
      KP = 1
      IPL = 0
      DO 32 I = 1,20
   32 IP2(I) = 0
      IP = 0
      READ(5,200) CASE
      WRITE(6,100) CASE,TIG,RE,GAMMA,ME,LIP,TE,AL,HO,VR,PHIU,XIU,TC,PHIR
     1,XIR,WLMIN,WLMAX,XM,IND,ISL,INT,LINES,NR,IPLOT
  200 FORMAT(10A8)
  100 FORMAT(1H1,3X,67HPROGRAM P2170      FLAME      ROCKET EXHAUST PLUME
     1OPTICAL SIGNATURES ///20X,10A8//11H INPUT DATA//29H TIME AFTER IGN
     2ITION-TIG(SEC)F10.4,11X,25HNOZZLE EXIT RADIUS-RE(CM)F10.4,13X, 5HG
     3AMMAF10.4/  X,19HEXIT MACH NUMBER-MEF10.4,20X,25HNOZZLE LIP ANGLE-
     4LIP(DEG)F10.4,13X,26HEFFECTIVE EARTH TEMP-TE(K)F8.2/  X,18HLOOK AN
     5GLE-AL(DEG)F8.2,23X,22HENGINE ALTITUDE-HO(FT)F9.0,17X,23HROCKET VE
     6LOCITY-VR(FPS)F9.2/  X,27HVERT. POLAR ANGLE-PHIU(DEG)F8.2,14X,30HV
     7ERT. AZIMUTHAL ANGLE-XIU(DEG)F8.2,10X,19HCHAMBER TEMP.-TC(K)F8.2/
     8 X,26HVEL. POLAR ANGLE-PHIR(DEG)F8.2,15X,29HVEL. AZIMUTHAL ANGLE-X
     9IR(DEG)F8.2/16H BAND LIMITS AREF6.2,3H TOF6.2,19X,30HRADIATING SPE
     FCIES MASS FRAC-XM F8.5,//                       23H CONTROL FLAGS F
     AOLLOW--//8H   IND =I5,25H (1 FOR DAY, 0 FOR NIGHT)/8H   ISL =I5,79
     BH (1,-1 FOR PARTICLES; 0,-2 FOR GAS ONLY; NEGATIVE TO SKIP INITIAL
     G CALCULATIONS)                         /8H   INT =I5,56H (APPROXIMATE
     CNUMBER OF INTEGRATION STEPS THROUGH LAYER)/8H LINES =I5,56H (APPRO
```

```
      OXIMATE NUMBER OF LINES OF SIGHT IN ONE DIMENSION)/8H     NR =I5,44H
      E (NUMBER OF ENTRIES IN FIRING HISTORY TABLE)/8H IPLOT =I5,42H (0 F
      FOR NO PLOT, 1 FOR ISOINTENSITY PLOTS))
C ***** INITIALIZATION OF SD 4020 CAMERA
      IF(IPLOT.NE.0) GO TO 30
   31 IR = (NR+2)/3
      K = 1
      DO 2 I = 1,IR
      DO 2 J =1,3
      L = I + (J-1)*IR
      TIDP(K) = TID(L)
      PDEP(K) = PDE(L)
    2 K = K +1
      WRITE (6,101)(TIDP(I),PDEP(I),I =1,NR)
  101 FORMAT(72H0 FIRING HISTORY TABLE FOLLOWS, EXIT-PLANE DENSITY (G/CM
     13) VS TIME (SEC) //3(4X,4HTIME,5X,7HDENSITY,5X)/(3(F10.4,E11.3,4X)
     2)
C
C ***** REGION 1,  INITIAL CALCULATIONS - PARTICLES  *****
C
      CC =(WLMAX - WLMIN)/2.
      DD =(WLMAX + WLMIN)/2.
      IF(ISL.NE.1) GO TO 3
      DO 4 I = 1,20
      T = TET(I)
C FIND WAVELENGTH WHERE PLANCK FUNCTION IS GREATEST. TRANSFORM INTEGRAL
      WW = 2898./T
      C = 3.9*WW
      D = 4.1*WW
C PERFORM GAUSSIAN INTEGRATION  (EQUATIONS 4-1,-2,-4,-5)
      BPASS(I) = 0.
      R1X = 0.
      DO 7 K = 1,NP
      EPT(I,K) = 0.
    7 ETL(I,K) = 0.
      DO 5 J = 1,8
      WL = C*XP(J) + D
      WWL = CC*XP(J) + DD
      R1 = 3.742E4/WL**5/(EXP(14390./WL/T)-1.)
      R2 = 3.742E4/WWL**5/(EXP(14390./WWL/T)-1.)
      R1X = R1X + W(J)*R1
      BPASS(I) = BPASS(I) + R2*W(J)
C SUM OVER PARTICLES
      DO 5 K = 1,NP
      EM1 = EMIT(RP(K),WL)
      EM2 = EMIT(RP(K),WWL)
      EPT(I,K) = EPT(I,K) + W(J)*R1*EM1
    5 ETL(I,K) = ETL(I,K) + W(J)*R2*EM2
      DO 6 K = 1,NP
      EPT(I,K) = EPT(I,K)/R1X
    6 ETL(I,K) = ETL(I,K)/BPASS(I)
    4 BPASS(I) = BPASS(I)*CC/3.14159
      WRITE(6,103) (RP(I),I=1,NP)
  103 FORMAT(46H0 INITIAL CALCULATIONS FOR PARTICLE PROPERTIES//35X,16HT
     1OTAL EMISSIVITY//16X,27HPARTICLE RADIUS (CM X 10E4)/13H   TEMP(K)
     2   ,4P10F10.3//)
      DO 20 I = 1,20
   20 WRITE(6,104) TET(I),(EPT(I,J),J=1,NP)
  104 FORMAT(1H ,F9.1,3X,10F10.5)
```

```
      WRITE(6,107) WLMIN,WLMAX,(RP(I),I=1,NP)
  107 FORMAT(////13HO BAND DATA,(,F5.1,4H TO ,F5.1,1H)//32X,18HEMISSIVIT
     1Y IN BAND//12X,42HPLANCK FUNC.   PARTICLE RADIUS (CM X 10E4)/25H
     2 TEMP(K)   (W/CM2/SR)    ,4P10F9.3//)
      DO 22 I = 1,20
   22 WRITE(6,108) TET(I),BPASS(I),(ETL(I,J),J = 1,NP)
  108 FORMAT(1H ,F9.1,F13.4,2X,10F9.5)
C COOLING CURVES FOR SOLID PARTICLES (SECTION 4.1.2,EQS. 4-3 AND 4-8)
      DT = (TF-100.)/19.
      TCOO(1) = TF
      DO 8 K = 1,NP
      TCIX(1,K) = 0.
      T1(K) =1./TF**4*RP(K)    /TABLE(TF,TET,EPT(1,K),20,0,1)
    8 DELT(K) = T1(K)*DF/1.7016E-11
      T = TF
      DO 9 I = 2,20
      T = T - DT
      TCOO(I) = T
      DO 9 K = 1,NP
      T2(K) = 1./T**4*RP(K)    /TABLE(T,TET,EPT(1,K),20,0,1)
      TCIX(I,K) = TCIX(I-1,K)+(T2(K) + T1(K))/2.*DT
    9 T1(K) = T2(K)
      DO 10 K = 1,NP
   10 TCIX(20,K) = TCIX(20,1)
C REARRANGE TO CHANGE INDEPENDENT VARIABLE TO TIME.
      DT = TCIX(20,1)/722700.
      TIME = -DT
      DO 11 I = 1,20
      TIME = TIME + DT*FLOAT(I**4)
      TCI(I) = TIME/1.7016E-11*CP
      DO 11 K = 1,NP
   11 TCOOL(I,K) =                TABLE(TIME,TCIX(1,K),TCOO,20,0,1)
      TCI(20) = 1.E5
      WRITE(6,105) (RP(I),I=1,NP)
  105 FORMAT(///25X,34HPARTICLE TEMPERATURE HISTORIES (K)//16X,27HPARTIC
     1LE RADIUS (CM X 10E4)/13H  TIME(SEC)  ,4P10F10.3//)
      DO 21 I = 1,20
   21 WRITE(6,106) TCI(I),(TCOOL(I,J),J = 1,NP)
  106 FORMAT(1H ,F10.3,2X,10F10.1)
    3 IF(ISL.LE.0) GO TO 15
      TO = 400.
      DO 12 K = 1,NP
      TCOOL(20,K) = 70.
C NEWTON-RAPHSON SOLUTION TO EQ. 4-7 (AND EQ. 4-6)
      C   = 5.964E10*TABLE(6000.,TET,EPT(1,K),20,0,1)*FLOAT(IND) +
     1TABLE(TE,TET,EPT(1,K),20,0,1)*TE**4/2.
      DO 13 I = 1,100
      TN = (2.*TO + C /TO**3/TABLE(TO,TET,EPT(1,K),20,0,1))/3.
      IF (ABS(TN-TO).LT..1) GO TO 14
   13 TO = TN
      WRITE (6,102) TN,TO,K
  102 FORMAT(58HOERROR, SOLUTION OF EQ.(4-7) DOES NOT CONVERGE. NEW VALU
     1E= G8.2, 12H, OLD VALUE= G8.2,I5,18H-TH PARTICLE GROUP)
   14 TMIN(K) = TN
   12 CONTINUE
      WRITE(6,109) (RP(I),I = 1,NP)
  109 FORMAT(////27H  PARTICLE RADIUS (CMX10E4) ,4P10F10.3)
      WRITE(6,110) (DELT(I),I = 1,NP)
  110 FORMAT(27H  SOLIDIFICATION TIME (SEC) ,10F10.3)
```

```
      WRITE(6,111) (TMIN(I),I = 1,NP)
  111 FORMAT(27H EQUILIBRIUM TEMPERATURE(K) ,10F10.2)
C
C ***** REGION 2, INITIAL CALCULATIONS - GAS  *****
C
C IF BAND LIMITS UNCHANGED FROM PREVIOUS RUN, SKIP BAND-ENERGY CALC.
   15 IF(ISL.LT.0) GO TO 16
C DETERMINE ET (EQ. 4-11)
      Q1 = 0.5E4/B/WLMAX
      Q2 = 0.5E4/B/WLMIN
      DO 18 I = 1,20
      T = TR(I)
      SIG = 1.4387*B/T
      XMIN = Q1*Q1*SIG
      XMAX = Q2*Q2*SIG
      ET(I) =                 9.928E-4*B/SIG*(Q1*EXP(-XMIN) - Q2*EXP(-XMAX
     1) + .88623/SQRT(SIG)*(ERF(Q2*SQRT(SIG)) - ERF(Q1*SQRT(SIG))))
C DETERMINE BAND RADIANCE (EQ. 4-15)
      JM(I) = 0.
      DO 17 J = 1,8
      WWL = CC*XP(J) + DD
   17 JM(I) = JM(I) + W(J)*ABCO(WWL,T)/WWL**5/(EXP(14390./WWL/T)-1.)
   18 JM(I) = JM(I)*CC*3.742E4/PI
      WRITE(6,112) (TR(I),ET(I),JM(I),I = 1,20)
  112 FORMAT(////43HOROTATIONAL RADIATION FROM WATER MOLECULES./97H
     1AVAILABLE BAND ENERGY = ET (10E-20 W SEC/MOLEC), BAND RADIATIVE RA
     2TE = J (10E-20 W/SR MOLEC) //24X,4HT(K),8X,2HET,12X,1HJ//(22X,F7.1
     3,2X,2E13.3))
C CALCULATION OF RADIATIVE COOLDOWN  (EQ. 4-16)  (J VS TIME)
      ETX = ET(1)
      TO = 0.
      DET = ET(20)/190.
      JT(1) = JM(1)
      TJ(1) = 0.
      DO 19 I = 2,20
      ETX = ETX +DET
      JT(I) = TABLE(ETX-ET,JM,20,0,1)
      TN = .079578/JT(I)
      TJ(I) = TJ(I-1) +(TO + TN)/2.*DET
   19 TO = TN
C FIT DATA WITH FUNCTION LINEAR IN TIME (EQ. 4-17)
      CALL LSQPOL(TJ,JT,WP,TIDP,20,ETX,2,CU)
      A =  CU(2)
      DO 23 I = 1,20
      TCOO(I) = CU(1) + CU(2)*TJ(I)
      TIDP(I) = TABLE(JT(I),JM,ET,20,0,1)
   23 PDEP(I) = TABLE(JT(I),JM,TR,20,0,-1)
      WRITE(6,113) (TJ(I),JT(I),TCOO(I),TIDP(I),PDEP(I),I = 1,20)
  113 FORMAT(////28H     RADIATIVE DECAY IN BAND //9X, 9HTIME(SEC),6X,1H
     1J,8X,6HJ(FIT),7X,2HET,8X,4HT(K)//(6X,F11.5,2X,4E11.3))
C SOLUTION FOR LIMITING VELOCITY (EQS. 4-9 AND 4-10)
   16 VP = 2.*SQRT(TC/WMOL/(2.-2./GAMMA)*8.3144E7)
C
C PREPARATION FOR PLUME INTEGRATION
C
      X1 = 0.
      Y1 = 0.
C CALCULATION OF MAXIMUM PLUME CONE HALF-ANGLE AND MAXIMUM PLUME RADIUS
      GG = SQRT((GAMMA+1.)/(GAMMA-1.))
```

```
      GM = SQRT(ME**2 - 1.)
      CONE = LIP +  RA*(GG-1) -        RD*(GG*ATAN(GM/GG) -ATAN(GM))
      RM = VP*TIG
      DELS = (RM + RM*COS(AL/RD))*.05/(1.1**(INT/2) - 1.)
      IA = 0.
      P1   = PHIR/RD
      P2   = PHIU/RD
      XI1 = XIR/RD
      XI2 = XIU/RD
      IF(AL.GT.45.) IA = 1.
      DELX = DELS*COS((RA*IA - AL)/RD)
      DELY = (RM + RM*SIN(AL/RD))*.1/(1.1**LINES - 1.)
      CD = COS((RA*IA-AL)/RD)
      DELXX = DELY/CD
      DEL = DELY
      J0 = 10000
C SELECTION OF ISOINTENSITY LEVELS TO BE PLOTTED
      FC(1) = 1.E-11
      X10  = SQRT(10.)
      DO 29 I = 1,19
   29 FC(I+1) = FC(I)*X10
C
C ***** REGION 3. SELECTION OF INITIAL VALUE OF X1 *****
C
      VV = VR*30.4801
      V1 = VV*SIN(P1)*COS(XI1)
      V2 = VV*SIN(P1)*SIN(XI1)
      V3 = VV*COS(P1)
      INOUT = 2
      IF(IA.EQ.1.) GO TO 36
      IF(XM.EQ.0.) GO TO 27
      IF(CONE.GT.90.) GO TO 25
      IF(90. - CONE.GE.AL) GO TO 27
   34 X1 = RM*COS((CONE + AL)/RD)/COS(AL/RD)
      GO TO 27
   25 IF(90. - CONE.GE.AL - 90.) GO TO 34
      X1 = -RM/COS(AL/RD)
      GO TO 27
   36 IF(XM.NE.0.) GO TO 33
      X1 = -RM*COS((AL - 33.52)/RD)/SIN(AL/RD)
      GO TO 27
   33 IF(CONE.LT.90..AND.CONE.LT.AL) GO TO 35
      X1 = -RM/SIN(AL/RD)
      GO TO 27
   35 X1 = -RM*COS((AL - CONE)/RD)/SIN(AL/RD)
   27 XPLOT = 0.
      PX(1) = XPLOT
      DELX1 = (DELXX + ABS(X1)/10.)/1.1
      R = 1./1.1
C PLOT LIMITS FOR GRID
      XL = 0.
      XR = RM - X1*COS((RA*IA - AL)/RD)
      IF(IA.EQ.1..AND.XM.EQ.0.) XR = 2.*RM*SIN(33.52/RD)
     1  *SIN(AL/RD)
      YB = 0.
      YT = XR
C
C  ***** REGION 4. CALCULATE INTEGRATION BOUNDARIES  *****
C
```

```
   24 CALL STAB(PMIN,PMAX,INOUT)
      DELY = DELY*1.1
      IF(INOUT.EQ.0) GO TO 26
      Y1 = 0.
      DELY = DEL
      JO =  MINO(JO,JP)
      KP = KP + 1
      IF(KP.EQ.4.AND.IPLOT.NE.0) CALL CONTUR(KP)
      JP = 0
      IF (X1.GE.0.) R = 1.1
      DELX1 = DELX1*R
      X1 = X1 + DELX1
      XPLOT = XPLOT + DELX1*CD
C STORE VALUES (PX,PL,PY) FOR PLOTTING
      PX(KP)=XPLOT
      CALL STAB(PMIN,PMAX,INOUT)
      IF(INOUT.EQ.0) GO TO 26
      GO TO 1
C
C ***** REGION 5, INITIAL EVALUATION OF INTEGRAND *****
C
   26 CALL FUN(SOLD,PMIN)
C
C ***** REGION 6, INTEGRATE THROUGH PLUME *****
C
      CALL SIMP(SOLD,SINT,PMAX,PMIN,DELX)
      SINT = SINT*.6666667
      WRITE(6,114) XPLOT,Y1,SINT
  114 FORMAT(4X,2F10.2,E11.3)
      JP = JP + 1
      PL(KP,JP)= SINT
      PY(JP) = Y1
      Y1 = Y1 + DELY
      GO TO 24
   30 IF(IPT.EQ.0) CALL CAMRAV(IPLOT)
      IPT = 1
      GO TO 31
      END
```

```
      SUBROUTINE SIMP(SOLD,SINT,PMAX,PMIN,DEL)
      SINT = SOLD
      DELX = ( ABS(PMIN) /10. + DEL)/1.1
      A = 1./1.1
      X = PMIN
    1 SUM = 0.
      C = 4.
      DELX = DELX*A
    4 X = X + DELX
      IF(X.GT.PMAX) GO TO 2
      CALL FUN(SS,X)
      SS = SS*DELX
      SUM = SUM + C*SS
      IF(C.EQ.4.) GO TO 3
      IF(X.GT.0.) GO TO 6
    5 SOLD = SS
      SINT = SINT + SUM
      GO TO 1
    3 C = 2.
      GO TO 4
    2 SINT = SINT - SOLD
      RETURN
    6 A = 1.1
      GO TO 5
      END
```

```
      SUBROUTINE STAB(PMIN,PMAX,INOUT)
      DIMENSION X(2)
      COMMON/STC/X,       XX
      COMMON/MSTCF/IA,AL,RD,CONE,RM,X1,Y1,RA
      REAL IA
      IF(INOUT.NE.2) GO TO 1
      A0 = ABS (TAN((RA*IA - AL)/RD))
      A1 = A0*A0
      A2 =  (TAN((RA*(IA-1.)- CONE)/RD))**2
    1 INOUT = 0
      B1 = X1*A0/(1. + A1)
      B = Y1**2
      C1 = B + X1**2     -RM*RM
      IF(IA.EQ.0.)B = -B*A2
      C2 = X1*X1 + B
      RAD = B1*B1 - C1/(1. + A1)
      II = 1
C   CHECK FOR LINE-OF-SIGHT BEYOND R=RP.
      IF(RAD.LT.0.) GO TO 5
      RAD = SQRT(RAD)
      XX = -B1 + RAD
      CALL CHECK(II)
      IF(II.GT.2) GO TO 4
      XX = -B1 - RAD
      CALL CHECK(II)
      IF(II.GT.2) GO TO 4
    3 BD = A1 - A2
      B2 = X1*A0/BD
C   CHECK FOR LINE-OF-SIGHT BEING PARALLEL TO PLUME BOUNDARY.
      IF(ABS(BD).LT.1.E-8) GO TO 2
      RAD = B2*B2 - C2/BD
      IF (RAD.LT.0.) GO TO 5
      RAD = SQRT(RAD)
      XX = -B2 + RAD
      CALL CHECK(II)
      IF(II.GT.2) GO TO 4
      XX = -B2 - RAD
    7 CALL CHECK(II)
      IF(II.GT.2) GO TO 4
    5 INOUT = 1
      RETURN
    2 IF(AL.EQ.0..OR.AL.EQ.RA) GO TO 5
      XX = C2/B2
      GO TO 7
    4 IF (X(1).GT. X(2)) GO TO 6
      PMIN = X(1)
      PMAX = X(2)
      RETURN
    6 PMIN = X(2)
      PMAX = X(1)
      RETURN
      END
```

```
      SUBROUTINE CHECK(I)
      DIMENSION X(2)
      COMMON/STC/X,      XX
      COMMON/MSTCF/IA,AL,RD,CONE,RM,X1,Y1,RA
      REAL IA
C  CHECK TO SEE IF XX REPRESENTS X OR Z.
      IF(IA.EQ.0.) GO TO 1
      ZZ = XX
      RR =     SQRT(Y1*Y1 + (ZZ*TAN((RA-AL)/RD)  + X1)**2 +ZZ*ZZ)
    2 IF (RR.EQ.0.) RETURN
      THETA = RD*ACOS(ZZ/RR)
C  CHECK TO SEE IF INTERSECTION IS ON CONICAL PLUME BOUNDARY.
      IF(ABS((THETA-CONE)/CONE).LT..001) GO TO 3
C INTERSECTION IS ON SPHERICAL CAP OF PLUME.
      IF(THETA.GT.CONE) RETURN
C DO THE FOLLOWING IF THIS POINT IS INDEED ONE OF THE INTEGRATION LIMITS
    4 X(I) = XX
      I = I+1
      RETURN
    1 ZZ =   XX*TAN(AL/RD) + X1
      RR = SQRT(Y1*Y1 + XX*XX + ZZ*ZZ)
      GO TO 2
C INTERSECTION IS ON CONICAL BOUNDARY
    3 IF(RR/RM.LT.1.001) GO TO 4
      RETURN
      END
```

```
      SUBROUTINE FUN(O,P)
      DIMENSION PDE(50),TID(50),TET(20),TCOOL(20,10),TCI(20),BPASS(20),
     1ETL(20,10),TMIN(10),DELT(10),ET(20),TR(20),RP(10),N(10),AB(10),B(1
     20),C(10),TH(10),D(10),E(10),F(10),HT(20),TMT(20),CAT(20),XP(8)
     3,W(8)
      REAL MF,IA,J,JG,N
      COMMON/ME/ISL,NP,R,RE,GAMMA,ME,AN,TIG,TID,PDE,NR,XML,DELT,TMIN,TCI
     1,TCOOL,PI,RP,TET,ETL,BPASS,TF,VP,PHIU,XIU,TR,ET,A,V1,V2,V3,HO,WMOL
     2,XM,XP,W,GL
      COMMON/MSTCE/IA,AL,RD,CONF,RM,X1,Y1,RA
      DATA WMOL,XML,DP/17. , .36    ,4.0045/,S,CF/.7E-15,3.531445E-5/
C CURVE FIT COEFFICIENT V AND RHO CORRELATIONS FOR TWO-PHASE FLOW
      DATA AB/2.6007E5,2.6764E5,2.7520E5,2.8248E5,2.8932E5,2.9701E5,3.07
     176E5/,B/-2.9745E4,-2.9337E4,-2.9355E4,-3.0367E4,-3.2891E4,-3.8094E
     24,-4.6692E4/,C/.02092,.02064,.02036,.02012,.01976,.01930,.01874/,N
     3/-.6425,-1.0308,-1.3175,-1.4886,-1.5419,-1.3494,-.9048/,D/.5396,.8
     487,1.3339,1.0308,.5395,.4053,.2201/,E/2.04,2.06,2.08,2.12,2.17,2.2
     54,2.33/,F/-2.05,6.30,6.95,7.30,6.75,5.65,2.84/,TH/13.56,14.75,16.8
     61,19.76,23.53,28.22,33.52/
C H(FT),T, AND LOG(N/FT3) FROM U.S.STANDARD ATMOSPHERE (1962)
      DATA HT,TMT,CAT/3.E5,3.5E5,4.E5,4.5E5,5.E5,5.5E5,6.E5,6.5E5,7.E5,8
     1.E5,9.E5,1.E6,1.1E6,1.3E6,1.5E6,1.7E6,1.9E6,2.1E6,2.3E6,3.0E6,184.
     295,241.74,385.34,663.20,924.16,1084.09,1170.47,1230.3,1275.21,1346
     3.67,1396.14,1435.68,1455.53,1485.69,1489.35,1496.29,1500.57,1506.3
     43,2*1507.58,18.15,16.98,16.05,15.41,15.01,14.75,14.55,14.36,14.20,
     513.90,13.64,13.40,13.17,12.76,12.41,12.08,11.76,11.49,11.20,10.3/
      X = P
      JG = 0.
      S1 = 0.
      G2 = GAMMA - 1.
      Z = X1 + ABS(TAN((RA*IA-AL)/RD))*X
      IF(IA.EQ.1.) GO TO 6
    7 R = SQRT(X*X + Y1*Y1 + Z*Z)
      AN = ACOS(Z/R)*RD
      IF(IABS(ISL).NE.1) GO TO 1
      G1 = (GAMMA + 1.)/2.
C CALCULATE THROAT RADIUS FROM EXIT RADIUS AND EXPANSION RATIO
      RT = RE/SQRT(G1**(-G1/G2)*(1.+G2*ME*ME/2.)**G1/G2/ME)
      CT= COS(AN/RD)
      DO 2 I = 1,NP
C CALCULATE PARTICLE TIME OF FLIGHT, EQ. 4-25
      IF(TH(I).LT.AN) GO TO 2
C ARE WE BEYOND THE LIMITING STREAMLINE FOR THIS SIZE PARTICLE
      TI =              R/CT**N(I)/(AB(I)+ B(I)*EXP(-C(I)*R/RT))
      IF(TI.GT.TIG) GO TO 2
      PDO = TABLE(TIG-TI,TID,PDE,NR,0,1)*XML
C PARTICLE CONCENTRATION
      PD = PDO*D(I)*(RT/R)**E(I)*CT**F(I)/4.18879/DP/RP(I)**3
C CHECK FOR INCOMPLETE SOLIDIFICATION OF PARTICLE
      IF(TI.LT.DELT(I)) GO TO 4
      T = AMAX1(TMIN(I),TABLE(TI -DELT(I),TCI,TCOOL(1,I),20,0,1))
    3 BO = TABLE(T,TET,BPASS      ,20,0,1)
C PARTICLE RADIANCE ACCUMULATION, EQ. 4-27
      S1 = S1 + PD*PI*RP(I)**2  *TABLE(T,TET,ETL(1,I),20,0,-1)*BO
    2 CONTINUE
      GO TO 1
    4 T = TF
      GO TO 3
C GAS-PHASE CALCULATIONS.
```

```
    1 TI= R/VP
      IF(TI.GT.TIG.OR.AN.GE.CONE) GO TO 5
      GDO = TABLE(TIG-TI,TID,PDE,NR,0,1)*6.0254E23/WMOL
      XK = GAMMA*G2*MF*MF
C GAS DENSITY, EQ 4-29 (MODIFIED FOR TWO-PHASE FLOW)
      GD = GDO*XK/2.*(RE/R)**2*XM*(COS(RA/CONE*AN/RD))**(XK-2.)
      IF(GD .LT.1.)GO TO 5
      VX = X*VP/R
      VZ = Z*VP/R
C RELATIVE VELOCITY, EQ 3-16
      VREL = SQRT((VX+V1)**2 + (VZ+V3)**2 + (Y1*VP/R +V2)**2)
C FIND ALTITUDE IN PLUME  EQ. 4-40
      H = HO +(SIN(PHIU)*(X*COS(XIU )+ Y1*SIN(XIU)) +Z*COS(PHIU))/30.48
      H = AMAX1(H,3.E5)
      H = AMIN1(H,3.E6)
C FIND EXCITATION TEMPERATURE, EQ. 4-35
      TA = TABLE(H,HT,TMT,20,0,1)
      TM = AMAX1(TA,VREL*(9.61194E-9*VREL + 1.07636E-3))
C ATMOSPHERIC CONCENTRATION
      CA =(EXP(TABLE(H,HT,CAT,20,0,-1)*GL))*CF
C CALCULATION OF ATTENUATION FACTOR, EQ. 2-7
      HD = (H - HO)/2.
      AT = 0.
      DO 8 I = 1,8
      HS = HO + HD*(XP(I) + 1.)
    8 AT = AT + W(I)*EXP(TABLE(HS,HT,CAT,20,0,1)*GL)
      AT = AT*R/2.*CF*S
C RADIATIVE RELAXATION TIME, EQ. 4-18 AND 4-36
      EM = TABLE(TM,TR,ET,20,0,1)
      TAU = SQRT(EM/2./PI/A)
C COMPUTATION OF EMISSION, EQ. 4-38 AND 4-39
      TX = 1. - TI/TAU
      J =  A*TAU/2.
      IF(TX.GT.0.) J = J + J*TX
C RADIANCE CALCULATION, EQ. 4-37
      JG = VREL*CA*S*TAU*(1.-EXP(-TI/TAU))*GD*J*1.E-20*EXP(-AT)
    5 Q = S1 + JG
      RETURN
    6 TX = Z
      Z = X
      X = TX
      GO TO 7
      END
```

```
      FUNCTION ABCO(WL,TE)
C   COMPUTES ABSORPTION COEFFICIENT PER MOLECULE  OF WATER IN THE
C    ROTATIONAL SPECTRUM, SCALED BY 10E20
      IF(T1.EQ.TE) GO TO 1
      T1 = TE
      T = SQRT(T1)
      WO = 10.6*T
      YO = 279.96/(T + 50.031) - 3.208
      A =  1.0303E-4 - 105.582E-4/(T-2.896)
    1 W = 10000./WL
      ABCO = EXP(2.302585*(YO + A*(W - WO)**2/SQRT(W)))*1.3803E-2*TE
      RETURN
      END
```

```
      FUNCTION EMIT(R,W)
C COMPUTES THE EMISSIVITY OF ALUMINA PARTICLES OF RADIUS R (CM) AT A
C WAVELENGTH W (MICRONS) BASED ON CORRELATION OF MIE CALCULATIONS.
      IF (A.EQ.0.) GO TO 1
      IF (R.EQ.RO) GO TO 2
    1 A = (11.32 -  2220.*R) *R + .00689
      RO = R
    2 EMIT = A*(W + 2.5)
      RETURN
      END
```

```
      SUBROUTINE CONTUR(KP)
      DIMENSION TT(9),XP(21),YP(21),PX(3),PY(100),PL(3,100),FC(20),CA(6)
     1,IP2(20)
      DATA TT/9*10H          /
      COMMON/MCG/ JO,IPL,PX,PL,PY,FC,XL,XR,YB,YT,IP,IP2
      JO =((JO + 1)/2)*2 - 1
      IF (IPL.EQ.0) CALL GRID
      J1 = JO - 2
      FD = 0.
      DX =(PX(3) - PX(1))/20.
      DO 1 L = 1, J1, 2
C CURVE FIT TO 3 X 3 MATRIX OF PLUME POINTS
      CALL CF(PX,PY(L),PL(1,L),CA)
C HERE TO STATEMENT 7 -- FINDS WHAT ISOINTENSITY LINES ARE IN MATRIX
      BIG = 0.
      SMALL = 1.E20
      L2 = L+2
      DO 2 I = 1,3
      DO 2 J = L,L2
      A = PL(I,J)
      IF(A.EQ.0.) GO TO 2
      BIG = AMAX1(BIG,A)
      SMALL = AMIN1(SMALL,A)
    2 CONTINUE
      YS = PY(L)
      YL = PY(L2)
      DO 9 I = 1,20
      IF (FC(I).LT.SMALL.OR.FC(I).GT.BIG) GO TO 9
      IF (FD.GT.FC(I)) GO TO 7
      FD = FC(I)
      ID = I
      N = L
C HERE TO STATEMENT 3 -- SOLVES FOR COORDINATES OF ISOINTENSITY LINES
    7 K = 0
      X =  PX(1)
      DO 3 J = 1,20
      B = CA(4) + CA(5)*X
      A = CA(3)
      C = CA(1)*X**2 + CA(2)*X - FC(I) + CA(6)
      D = B**2 - 4.*A*C
      IF (D.LT.0.) GO TO 4
      D = SQRT(D)
      Y = (-B + D)/2./A
      IF(Y.GT.YS.AND.Y.LT.YL) GO TO 5
      Y = (-B - D)/2./A
      IF(Y.GT.YS.AND.Y.LT.YL) GO TO 5
    4 X = X + DX
    3 CONTINUE
      IF (K.LE.1) GO TO 9
      CALL AICRT3(0,0,XP,YP,K,1,1,1,58,TT,TT,TT,2,1,16.,16.,2,XL,XR,2,
     1YB,YT)
    9 CONTINUE
    1 CONTINUE
      DO 6 M = 1,IP
    6 IF(IP2(M).EQ.ID) GO TO 10
      IP = IP + 1
      IP2(IP) = ID
      WRITE (6,100)  FD   ,PX(1),PY(N)
  100 FORMAT(22H ISOINTENSITY LINE FOR E9.2,18HW/CM2/SR IS NEAR (E9.2,1H
```

```
     1,E9.2,1H))
C  INITIALIZATION FOR NEXT GROUP OF 3X3 MATRICES
   10 PX(1) = PX(3)
      DO 8 I = 1,JO
    8 PL(1,I) = PL(3,I)
      KP = 2
      JO = 10000
      RETURN
    5 K = K + 1
      XP(K) = X
      YP(K) = Y
      GO TO 4
      END
```

```
      SUBROUTINE GRID
C THIS ROUTINE PREPARES THE GRAPH LIMITS FOR PLOTTING ISOINTENSITY LINES
      DIMENSION TITLE(8),XT(9),YT(9),PX(3),PY(100),PL(3,100),FC(20)
     1,IP2(20)
      COMMON/MCG/ JO,IPL,PX,PL,PY,FC,XL,XR,YB,Y2,IP,IP2
      DATA TITLE/3*1H ,6HPLUME ,6HCONTOU,2HRS/,XT/4*1H ,6HX (CM)/,
     1 YT/4*1H ,6HY (CM)/
      CALL AICRT3(0,0,X,Y,2,3,1,1,42,TITLE,XT,YT,1,1,16.,16.,2,XL,XR,2,
     1YB,Y2)
      IPL = 1
      RETURN
      END
```

```
      SUBROUTINE CF(X,Y,F,C)
C THIS ROUTINE PERFORMS A LEAST-SQUARE CURVE FIT TO 9 (X,Y) POINTS OF
C FUNCTION F, WHERE
C        F(FIT) = C(1)X**2 + C(2)X + C(3)Y**2 + C(4)Y + C(5)XY + C(6)
      DIMENSION X(3),Y(3),F(3,3),C(6),A(6,6),RT(18),IT(18)
      DO 1 I = 1,6
      C(I) = 0.
      DO 1 J = 1,6
    1 A(I,J) = 0.
      DO 2 I = 1,3
      X2 = X(I)**2
      A(1,1) = A(1,1) + X2*X2
      A(1,2) = A(1,2) + X2*X(I)
      A(2,6) = A(2,6) + X(I)
    2 A(2,2) = A(2,2) + X2
      DO 3 I = 1,3
      Y2 = Y(I)**2
      A(3,3) = A(3,3) + Y2*Y2
      A(3,4) = A(3,4) + Y2*Y(I)
      A(4,6) = A(4,6) + Y(I)
    3 A(4,4) = A(4,4) + Y2
      A(1,1) = 3.*A(1,1)
      A(1,2) = 3.*A(1,2)
      A(2,2) = 3.*A(2,2)
      A(3,3) = 3.*A(3,3)
      A(3,4) = 3.*A(3,4)
      A(4,4) = 3.*A(4,4)
      A(4,6) = 3.*A(4,6)
      A(2,6) = 3.*A(2,6)
      A(2,1) = A(1,2)
      A(4,3) = A(3,4)
      DO 4 I = 1,3
      DO 4 J = 1,3
      X2 = X(I)**2
      Y2 = Y(J)**2
      XY = X(I)*Y(J)
      A(1,3) = A(1,3) + X2*Y2
      A(1,4) = A(1,4) + X2*Y(J)
      A(1,5) = A(1,5) + X2*XY
      A(2,3) = A(2,3) + XY*Y(J)
      A(2,4) = A(2,4) + XY
      A(3,5) = A(3,5) + XY*Y2
      C(1) = C(1) + F(I,J)*X2
      C(2) = C(2) + F(I,J)*X(I)
      C(3) = C(3) + F(I,J)*Y2
      C(4) = C(4) + F(I,J)*Y(J)
      C(6) = C(6) + F(I,J)
    4 C(5) = C(5) + F(I,J)*XY
      A(3,1) = A(1,3)
      A(3,2) = A(2,3)
      A(4,1) = A(1,4)
      A(4,2) = A(2,4)
      A(4,5) = A(2,3)
      A(5,4) = A(2,3)
      A(5,1) = A(1,5)
      A(5,2) = A(1,4)
      A(2,5) = A(1,4)
      A(5,3) = A(3,5)
      A(6,6) = 9.
```

```
      A(1,6) = A(2,2)
      A(6,1) = A(2,2)
      A(6,2) = A(2,6)
      A(3,6) = A(4,4)
      A(6,3) = A(4,4)
      A(6,4) = A(4,6)
      A(5,5) = A(1,3)
      A(5,6) = A(2,4)
      A(6,5) = A(2,4)
      CALL SID(A,6,6,6,C,1,1,S,IR,RT,IT,SC)
      IF(S.LT.3.) WRITE(6,100) X(1),Y(1)
  100 FORMAT (38H CURVEFIT COEFFICIENTS INACCURATE AT (E9.2,2H, E9.2,1H)
     1)
      RETURN
      END
```

```
      FUNCTION ERF(ETA)
C     SEE HANDBOOK OF MATHEMATICAL FUNCTIONS WITH FORMULAS, GRAPHS AND
C     MATHEMATICAL TABLES. NBS, APPLIED MATHEMATICAL SERIES 55,
C     PAGE 299,  SECTION 7.1.26 (1964)
      DATA P/ 0.3275911/
      DATA A1,A2,A3,A4,A5 / 0.254829592,-0.284496736,1.421413741,
     1-1.453152027,1.06140543 /
      T = 1.0/(1. + P*ABS(ETA))
      S = T*(A1 + T*(A2 + T*(A3 + T*(A4 + A5*T))))
      ERF = 1. - S*EXP(-ETA**2)
      IF (ETA.LT. 0.0) ERF = -ERF
      RETURN
      END
```

```
      FUNCTION TABLE(XX,X,FX,MO,L,IEX)
      DIMENSION X(MO),FX(MO)
      M=MO
      IF (XX-X(M)) 1,1,3
    1 IF (XX-X(1)) 2,6,6
    2 TABLE=FX(1)
      GO TO 4
    3 TABLE=FX(M)
    4 WRITE(6,5)
    5 FORMAT(1H0,10X,38H*** ERROR, RANGE OF TABLE EXCEEDED *** )
      WRITE (6,101) XX,X(1),X(M),MO
  101 FORMAT (  3E13.6,I10)
      IF (IEX.EQ.0) GO TO 100
      STOP
    6 IF (IEX.LT.0) GO  TO 14
      M1 = 1
      M3=M
      M2=M/2
    7 IF (X(M2)-XX) 9,8,8
    8 M3=M2
      GO TO 10
    9 M1=M2
   10 J=(M1+M3)/2
      IF (J.EQ.M2) GO TO 11
      M2=J
      GO TO 7
   11 IF (X(J)-XX) 12,12,13
   12 M3=J+1
      M1=J
      GO TO 14
   13 M3=J
      M1=J-1
   14 IF (L.NE.0) GO TO 15
      TABLE=FX(M1)+(XX-X(M1))*(FX(M3)-FX(M1))/(X(M3)-X(M1))
      GO TO 100
   15 IF (M1.GT.1) GO TO 16
      I1=1
      I2=2
      I3=3
      GO TO 17
   16 IF (M.GT.M3) GO TO 18
      I3=M
      I2=M-1
      I1=M-2
   17 DF32=FX(I3)-FX(I2)
      DF21=FX(I2)-FX(I1)
      DENOM=(X(I3)-X(I2))*(X(I3)-X(I1))*(X(I2)-X(I1))
      A=((X(I2)-X(I1))*DF32-(X(I3)-X(I2))*DF21)/DENOM
      B=((X(I2)**2-X(I1)**2)*DF32-(X(I3)**2-X(I2)**2)*DF21)/DENOM
      TABLE=FX(I3)-A*(X(I3)**2-XX**2)+B*(X(I3)-XX)
      GO TO 100
   18 I4=M3+1
      I3=M3
      I2=M1
      I1=M1-1
      DF43=FX(I4)-FX(I3)
      DF32=FX(I3)-FX(I2)
      DF21=FX(I2)-FX(I1)
      A1=(X(I3)-X(I2))*(X(I3)-X(I1))*(X(I2)-X(I1))*DF43
```

```
      A2=(X(I4)-X(I3))*(X(I2)-X(I1))*(X(I4)+X(I3)-X(I2)-X(I1))*DF32
      A3=(X(I4)-X(I3))*(X(I4)-X(I2))*(X(I3)-X(I2))*DF21
      DENOM=(X(I4)-X(I3))*(X(I4)-X(I2))*(X(I4)-X(I1))*
     1        (X(I3)-X(I2))*(X(I3)-X(I1))*(X(I2)-X(I1))
      A=(A1-A2+A3)/DENOM
      B1=(X(I3)-X(I2))*DF43-(X(I4)-X(I3))*DF32
      B2=(X(I4)-X(I3))*(X(I3)-X(I2))*(X(I4)-X(I2))
      B=B1/B2-A*(X(I4)+X(I3)+X(I2))
      C=DF43/(X(I4)-X(I3))-A*(X(I4)**2+X(I3)**2+X(I4)*X(I3))
     1   -B*(X(I4)+X(I3))
      D=FX(I4)-(A*X(I4)**2+B*X(I4)+C)*X(I4)
      TABLE=A*XX**3+B*XX**2+C*XX+D
  100 RETURN
      END
```

```
 $IN TIG= 1.,RF = 5.7,GAMMA = 1.33, ME = 4.,LIP =15.,TE = 250.,AL =  0.,
 H0= 5.E5, VR = 2.E4,PHIU = 90.,XIU = 90.,PHIR = 90.,XIR = 90.,TC=1255.,
 WLMIN=8.,WLMAX=14., IND =0, ISL =1, INT = 20,  LINES =   30, NR = 5,
 TID= -.1,0.,2.,2.1,10.,PDE = 0.,2*1.1E-3,2*0., IPLOT = 1,XM = 0.0  $
BROADSIDE LOOK, PARTICLE ONLY, 600 LB THRUST, AT NIGHT
 $IN END = 1$
```

Appendix B
OUTPUT FOR SAMPLE PROBLEM

```
CORE MAP  17.52.37.  NORMAL                CONTROL                                                      000100    055005    000000    000000
       ---TIME---LOAD MODE --L1--L2----TYPE--------------USER-------CALL-------------FWA LOAD--LWA LOAD--BLNK COMN--LENGTH--
       FWA LOADER  073771  FWA TABLES  066023
       -PROGRAM----ADDRESS-                          --LABELED---COMMON--
       P2170       002247                                        MCG          000100
                                                                 MSTCF        001002
                                                                 MF           001012
       GRID        016337                                        MCG          000100
       CF          016441
       SIMP        017110
       ASCO        017216
       EMIT        017324
       STAB        017366                                        STC          017363
                                                                 MSTCF        001002
       CHECK       017622                                        STC          017363
                                                                 MSTCF        001002
       FUN         017741                                        MF           001012
                                                                 MSTCF        001002
       ERF         021066
       TABLE       021160
       CONTUR      022025                                        MCG          000100
       ACSHER      022504
       EXP         022544
       SINCOS      022575
       TAN         022674
       ALNLOG      023005
       ASINCOS     023074
       ATAN        023232
       RBAIEX      023324
       PBAREX      023367
       BUFFEO      023446
       GETBA       023552
       CAMRAV      023571
       CHSIZV      023737
       FLAGSV      024537                                        BUFFRV       023764
       HOLLV       024604
       INTBCD      024665
       LINEV       024711
       PLOTDD      025303                                        BUFFRV       023764
       PRINTV      025433
       RITS2V      025645
       RITSTV      026211
       SETCIV      026243
       SETMIV      026320
       SHIFTV      026412
       TABLIV      026435
       XAXISV      026731
       CHEKIT      027141
       LSQPOL      030335                                        ZZLSQP       027173
       MSGWTR      030526
       ORPOLC      030563
       ORPOLG      030742
       SIO         031635
       VLV         033013
       SQRT        033021
       INPUTC      033066
```

```
IOCHEK      033172
OUTPTC      033272
KOVALL      033376
KODER       033443
SIDS        034651
INPUTN      036231
SYSTEM      037513                     SCOPE2      037513
ASCII       040640
MDID        040667
DXDYV       042462
AICRT3      043032
KRAKER      044736
LOCF        045777
DABS        046002
DBAIEX      046017
OUTPTS      046074
ENDFIL      046160
APLOTV      046234
CNTIBM      046405
CTL4V       046515
ERRLNV      046707
ERRNLV      047012
FORMV       047117
FRAMEV      047272
GRIDIV      047477
HOLDIV      050516
LINRV       050545
LXYV        051162
NASORT      051212
NONLNV      051276
MXV         051635
OPMSGV      052036
PLOTND      052071
PLOTV       052344
POINTV      052466
SCERRV      052634
VCHARV      052656
VECTRV      053223
XHOOV       053252
XCHARV      053307
XSCALV      053331                     BUFFRV      023764
LABLV       053534
FN3CDV      054173
CNTCDC      054335
ERMRKV      054457
NOJTV       054506
PAGE4C      054607
SCOJTV      054745
----UNSATISFIED EXTERNALS-----                      REFERENCES
TOO FEW CONSTANTS FOR UNSUBSCRIPTED ARRAY
ERROR NUMBER   0149 DETECTED BY INPUTN  AT ADDRESS 036654
CALLED FROM P217     AT 002260
TOO FEW CONSTANTS FOR UNSUBSCRIPTED ARRAY
ERROR NUMBER   0149 DETECTED BY INPUTN  AT ADDRESS 036654
CALLED FROM P217C    AT 002260
```

```
PROGRAM P2170      FLAME      ROCKET EXHAUST PLUME OPTICAL SIGNATURES

                BROADSIDE LOOK, PARTICLE ONLY, 600 LB THRUST, AT NIGHT

INPUT DATA

TIME AFTER IGNITION-TIG(SEC)      1.0000     NOZZLE EXIT RADIUS=RE(CM)      5.7000     GAMMA      1.3300
EXIT MACH NUMBER=ME      4.0000              NOZZLE LIP ANGLE=LIP(DEG)      15.0000    EFFECTIVE EARTH TEMP=TE(K)   250.00
LOOK ANGLE=AL(DEG)      0.00                 ENGINE ALTITUDE=HO(FT)      500000        ROCKET VELOCITY=VR(FPS) 20000.00
VERT. POLAR ANGLE=PHIU(DEG)      90.00       VERT. AZIMUTHAL ANGLE=XIU(DEG)      90.00 CHAMBER TEMP.=TC(K) 1255.00
VEL. POLAR ANGLE=PHIR(DEG)      90.00        VEL. AZIMUTHAL ANGLE=XIR(DEG)      90.00
BAND LIMITS ARE  8.00 TO 14.00               RADIATING SPECIES MASS FRAC=XM 0.00000

CONTROL FLAGS FOLLOW--

  IND =    0 (1 FOR DAY, 0 FOR NIGHT)
  ISL =    1 (1,-1 FOR PARTICLES; 0,-2 FOR GAS ONLY; NEGATIVE TO SKIP INITIAL CALCULATIONS)
  INT =   20 (APPROXIMATE NUMBER OF INTEGRATION STEPS THROUGH LAYER)
LINES =   30 (APPROXIMATE NUMBER OF LINES OF SIGHT IN ONE DIMENSION)
   NR =    5 (NUMBER OF ENTRIES IN FIRING HISTORY TABLE)
IPLOT =    1 (0 FOR NO PLOT, 1 FOR ISOINTENSITY PLOTS)

 FIRING HISTORY TABLE FOLLOWS, EXIT-PLANE DENSITY (G/CM3) VS TIME (SEC)

   TIME      DENSITY          TIME      DENSITY          TIME      DENSITY
  -.1000   0.                2.0000   1.100E-03        10.0000   0.
   0.0000  1.100E-03         2.1000   0.

 INITIAL CALCULATIONS FOR PARTICLE PROPERTIES

                              TOTAL EMISSIVITY
```

PARTICLE RADIUS (CM X 10E4)

TEMP(K)	4.660	4.000	3.332	2.660	2.000	1.330	.690
99.0	.61595	.58284	.54672	.51338	.47762	.44027	.40361
150.0	.41620	.39409	.37101	.34711	.32293	.29768	.27289
200.0	.31945	.30249	.28476	.26642	.24786	.22848	.20946
250.0	.26140	.24752	.23302	.21801	.20282	.18696	.17140
300.0	.22270	.21087	.19[illegible]52	.18574	.17280	.15929	.14602
450.0	.19506	.18470	.17388	.16268	.15135	.13952	.12790
400.0	.17433	.16507	.15540	.14539	.13526	.12469	.11430
450.0	.15920	.1498[illegible]	.14103	.13194	.12275	.11315	.10373
500.0	.14530	.13759	.12953	.12119	.11274	.10393	.09527
600.0	.12596	.11926	.11228	.10505	.09773	.09009	.08259
700.0	.11213	.10617	.09996	.09352	.08701	.08020	.07352
800.0	.10177	.09636	.09072	.08488	.07896	.07279	.06673
1000.0	.08726	.08262	.07778	.07277	.06770	.06241	.05721
1200.0	.07756	.07344	.06916	.06470	.06020	.05549	.05087
1400.0	.07067	.06692	.06300	.05894	.05483	.05055	.04634
1700.0	.06335	.05998	.05648	.05284	.04916	.04531	.04154
2000.0	.05823	.05511	.05191	.04857	.04518	.04165	.03818
2500.0	.05243	.04964	.04674	.04373	.04068	.03750	.03438
3200.0	.04735	.04483	.04221	.03949	.03674	.03387	.03105
4001.0	.03388	.03682	.03466	.03243	.03017	.02781	.02549

BAND DATA,(8.0 TO 14.0)

EMISSIVITY IN BAND

TEMP(K)	PLANCK FUNC. (W/CM2/SR)	PARTICLE RADIUS (CM X 10E4) 4.660	4.000	3.332	2.660	2.000	1.330	.690
99.0	1.2463E-06	.17608	.16672	.15696	.14685	.13662	.12594	.11545
150.0	7.3850E-05	.16838	.15944	.15010	.14043	.13065	.12043	.11040
200.0	6.0061E-04	.16264	.15419	.14516	.13581	.12635	.11647	.10677
250.0	2.2275E-03	.15991	.15056	.14174	.13261	.12338	.11373	.10426
300.0	5.4898E-03	.15613	.14802	.13936	.13038	.12130	.11181	.10250
350.0	1.0638E-02	.15441	.14621	.13765	.12878	.11981	.11044	.10124
400.0	1.7685E-02	.15300	.14487	.13639	.12760	.11871	.10943	.10032
450.0	2.6500E-02	.15193	.14386	.13543	.12671	.11788	.10866	.09961
500.0	3.6897E-02	.15139	.14307	.13469	.12601	.11723	.10807	.09907
600.0	6.1644E-02	.14919	.14193	.13362	.12501	.11630	.10721	.09828
700.0	9.0503E-02	.14968	.14116	.13290	.12434	.11567	.10662	.09775
800.0	1.2239E-01	.14850	.14061	.13238	.12385	.11523	.10622	.09737
1000.0	1.9236E-01	.14774	.13989	.13170	.12321	.11463	.10567	.09687
1200.0	2.6768E-01	.14726	.13944	.13127	.12282	.11426	.10533	.09656
1400.0	3.4625E-01	.14673	.13913	.13098	.12254	.11401	.10509	.09634
1700.0	4.6794E-01	.14661	.13882	.13069	.12227	.11375	.10486	.09613
2000.0	5.9247E-01	.14639	.13861	.13049	.12209	.11358	.10470	.09598
2500.0	8.0365E-01	.14615	.13838	.13028	.12189	.11340	.10453	.09582
3200.0	1.1034E+00	.14594	.13819	.13010	.12172	.11324	.10439	.09569
6001.0	2.3194E+00	.14563	.13789	.12981	.12145	.11299	.10416	.09548

PARTICLE TEMPERATURE HISTORIES (K)

TIME(SEC)	PARTICLE RADIUS (CM X 10E4) 4.660	4.000	3.332	2.660	2.000	1.330	.690
0.000	2323.0	2323.0	2323.0	2323.0	2323.0	2323.0	2323.0
.002	2264.1	2280.1	2274.5	2266.1	2252.6	2222.5	2160.2
.014	2107.5	2087.4	2064.6	2030.4	1974.9	1883.6	1690.2
.052	1759.9	1724.3	1681.2	1617.8	1534.7	1404.3	1177.4
.145	1364.6	1326.4	1273.2	1214.4	1130.1	1009.0	814.2
.336	1032.7	1001.1	957.5	901.7	833.3	736.9	583.3
.691	791.2	765.6	727.9	680.4	631.7	551.0	436.9
1.296	622.2	595.[illegible]	562.6	534.6	489.1	429.0	331.4
2.266	497.0	473.3	446.6	426.4	393.6	332.2	279.8
3.744	4[illegible]8.3	392.2	369.8	336.3	319.1	287.7	216.7
5.907	329.4	322.1	311.9	296.7	272.1	222.5	214.2
8.972	292.6	281.9	266.0	242.9	216.4	213.7	210.7
13.192	241.9	225.8	215.8	213.7	211.3	208.7	206.0
18.869	212.4	210.7	205.8	206.8	204.6	202.2	199.5
26.391	2[illegible]2.8	201.2	199.5	197.7	195.7	193.5	191.1
36.036	190.4	189.1	187.6	185.9	184.2	184.2	180.1
48.378	174.7	173.9	172.3	170.9	169.5	167.9	166.1
63.892	154.8	154.[illegible]	153.1	152.1	151.0	149.8	148.6
83.151	130.2	129.[illegible]	129.3	128.7	12[illegible].1	127.5	126.8
100000.000	[illegible]	100.[illegible]	10[illegible].0	100.0	[illegible]	100.0	100.0

PARTICLE RADIUS (CMX10E4)	4.663	4.000	3.332	2.660	2.000	1.330	.690
SOLIDIFICATION TIME (SEC)	.073	[illegible]	.059	.050	.040	.029	.017
EQUILIBRIUM TEMPERATURE(K)	199.92	199.92	199.92	199.92	199.92	199.92	199.92

ROTATIONAL RADIATION FROM WATER MOLECULES.
AVAILABLE BAND ENERGY = ET (10E-20 J SEC/MOLEC), BAND RADIATIVE RATE = J (10E-20 W/SR MOLEC)

T(K)	ET	J
100.0	2.724E-84	3.313E-25
400.0	9.408E-15	4.554E-07
800.0	6.744E-10	2.197E-03
1200.0	3.223E-06	5.185E-02
1500.0	2.437E-04	2.807E-01
2000.0	3.447E-03	8.136E-01
2400.0	2.091E-02	1.700E+00
2700.0	5.796E-02	2.586E+00
3000.0	1.327E-01	3.634E+00
3300.0	2.638E-01	4.808E+00
3600.0	4.719E-01	6.070E+00
3900.0	7.776E-01	7.384E+00
4200.0	1.200E+00	8.715E+00
4500.0	1.758E+00	1.004E+01
5000.0	3.031E+00	1.216E+01
5500.0	4.781E+00	1.414E+01
6000.0	7.049E+00	1.593E+01
7000.0	1.322E+01	1.896E+01
9000.0	3.205E+01	2.343E+01
11000.0	5.836E+01	2.715E+01

RADIATIVE DECAY IN BAND

TIME(SEC)	J	J(FIT)	ET	T(K)
0.00000	3.313E-25	5.543E+00	2.724E-84	1.000E+02
.00241	5.071E+00	6.109E+00	3.072E-01	3.362E+03
.00665	6.682E+00	7.096E+00	6.144E-01	3.740E+03
.01004	7.837E+00	7.885E+00	9.215E-01	4.002E+03
.01299	8.783E+00	8.572E+00	1.229E+00	4.215E+03
.01567	9.511E+00	9.193E+00	1.536E+00	4.380E+03
.01815	1.018E+01	9.773E+00	1.843E+00	4.533E+03
.02050	1.069E+01	1.032E+01	2.150E+00	4.654E+03
.02273	1.121E+01	1.084E+01	2.457E+00	4.775E+03
.02486	1.172E+01	1.134E+01	2.765E+00	4.895E+03
.02691	1.221E+01	1.181E+01	3.072E+00	5.012E+03
.02888	1.256E+01	1.227E+01	3.379E+00	5.100E+03
.03080	1.290E+01	1.272E+01	3.686E+00	5.187E+03
.03267	1.325E+01	1.315E+01	3.993E+00	5.275E+03

.03449	1.340E+01	1.354E+01	4.301E+00	5.363E+03
.03627	1.394E+01	1.397E+01	4.608E+00	5.451E+03
.03800	1.424E+01	1.437E+01	4.915E+00	5.530E+03
.03971	1.449E+01	1.473E+01	5.222E+00	5.597E+03
.04138	1.473E+01	1.514E+01	5.529E+00	5.665E+03
.04303	1.497E+01	1.556E+01	5.836E+00	5.733E+03

0.00	0.00	0.
0.00	1487.44	0.
0.00	3123.62	0.
0.00	4923.42	0.
0.00	6903.21	0.
0.00	9080.97	0.
0.00	11476.50	0.
0.00	14111.59	0.
0.00	17010.19	0.
0.00	20198.65	0.
0.00	23705.95	0.
0.00	27563.99	0.
0.00	31807.83	0.
0.00	36476.05	0.
0.00	41611.09	0.
0.00	47259.64	0.
0.00	53473.04	0.
0.00	60307.79	0.
0.00	67826.01	0.
0.00	76096.05	0.
0.00	85193.09	0.
0.00	95199.84	0.
0.00	106207.26	0.
0.00	118315.43	0.
0.00	131634.41	0.
0.00	146285.29	0.
0.00	162401.26	0.
0.00	180128.82	0.
0.00	199629.14	0.
0.00	221079.50	0.
1352.22	0.00	0.
1352.22	1352.22	0.
1352.22	2839.66	0.
1352.22	4475.84	0.
1352.22	6275.64	0.
1352.22	8255.42	0.
1352.22	10433.18	0.
1352.22	12828.72	0.
1352.22	15463.81	0.
1352.22	18362.41	0.
1352.22	21550.87	0.
1352.22	25058.17	0.
1352.22	28916.21	0.
1352.22	33160.04	0.
1352.22	37828.27	0.
1352.22	42963.31	0.
1352.22	48611.86	0.
1352.22	54825.26	0.
1352.22	61660.00	0.
1352.22	69178.22	0.

1392.22	77448.26	0.
1392.22	86545.31	0.
1392.22	96552.05	0.
1392.22	107559.48	0.
1392.22	119667.64	0.
1392.22	132986.63	0.
1392.22	147637.51	0.
1392.22	163753.47	0.
1392.22	181461.04	0.
1392.22	200981.36	0.
2839.66	0.00	0.
2839.66	1352.22	0.
2839.66	2839.66	0.
2839.66	4475.84	0.
2839.66	6275.64	0.
2839.66	8255.42	0.
2839.66	10433.18	0.
2839.66	12828.72	0.
2839.66	15463.31	0.
2839.66	18362.41	0.
2839.66	21550.87	0.
2839.66	25058.17	0.
2839.66	28916.21	0.
2839.66	33160.04	0.
2839.66	37828.27	0.
2839.66	42963.31	0.
2839.66	48611.86	0.
2839.66	54825.26	0.
2839.66	61660.00	0.
2839.66	69178.22	0.
2839.66	77448.26	0.
2839.66	86545.31	0.
2839.66	96552.05	0.
2839.66	107559.48	0.
2839.66	119667.64	0.
2839.66	132986.63	0.
2839.66	147637.51	0.
2839.66	163753.47	0.
2839.66	181461.04	0.
2839.66	200981.36	0.
4475.84	0.00	0.
4475.84	1352.22	0.
4475.84	2839.66	0.
4475.84	4475.84	0.
4475.84	6275.64	0.
4475.84	8255.42	0.
4475.84	10433.18	0.
4475.84	12828.72	0.
4475.84	15463.81	0.
4475.84	18362.41	0.
4475.84	21550.87	0.
4475.84	25058.17	0.
4475.84	28916.21	0.
4475.84	33160.04	0.
4475.84	37828.27	0.
4475.84	42963.31	0.

4475.84	48611.86	0.
4475.84	54825.26	0.
4475.84	61660.00	0.
4475.84	69178.22	0.
4475.84	77448.26	0.
4475.84	86545.31	0.
4475.84	96552.05	0.
4475.84	107559.48	0.
4475.84	119667.64	0.
4475.84	132986.63	0.
4475.84	147637.51	0.
4475.84	163753.47	0.
4475.84	181481.04	0.
4475.84	200981.36	0.
6275.64	0.00	0.
6275.64	1352.22	0.
6275.64	2839.66	0.
6275.64	4475.84	0.
6275.64	6275.64	0.
6275.64	8255.42	0.
6275.64	10433.18	0.
6275.64	12828.72	0.
6275.64	15463.81	0.
6275.64	18362.41	0.
6275.64	21550.87	0.
6275.64	25058.17	0.
6275.64	28916.21	0.
6275.64	33160.04	0.
6275.64	37828.27	0.
6275.64	42963.31	0.
6275.64	48611.86	0.
6275.64	54825.26	0.
6275.64	61660.00	0.
6275.64	69178.22	0.
6275.64	77443.26	0.
6275.64	86545.31	0.
6275.64	96552.05	0.
6275.64	107559.48	0.
6275.64	119667.64	0.
6275.64	132986.63	0.
6275.64	147637.51	0.
6275.64	163753.47	0.
6275.64	181481.04	0.
6275.64	200951.36	0.
8255.42	0.00	0.
8255.42	1352.22	0.
8255.42	2837.66	0.
8255.42	4475.84	0.
8255.42	6275.64	0.
8255.42	8255.42	0.
8255.42	10433.18	0.
8255.42	12828.72	0.
8255.42	15463.81	0.
8255.42	18362.41	0.
8255.42	21550.87	0.
8255.42	25053.17	0.

8295.42	28916.21	0.
8295.42	33160.04	0.
8295.42	37828.27	0.
8295.42	42963.31	0.
8295.42	48611.86	0.
8295.42	54825.26	0.
8295.42	61660.00	0.
8295.42	69178.22	0.
8295.42	77448.26	0.
8295.42	86545.31	0.
8295.42	96552.05	0.
8295.42	107559.48	0.
8295.42	119667.64	0.
8295.42	132986.63	0.
8295.42	147637.51	0.
8295.42	163753.47	0.
8295.42	181481.04	0.
8295.42	200981.36	0.
10433.18	0.00	2.514E-06
10433.18	1352.22	2.437E-06
10433.18	2839.66	0.
10433.18	4475.84	0.
10433.18	6275.64	0.
10433.18	8255.42	0.
10433.18	10433.18	0.
10433.18	12828.72	0.
10433.18	15463.81	0.
10433.18	18362.41	0.
10433.18	21550.87	0.
10433.18	25058.17	0.
10433.18	28916.21	0.
10433.18	33160.04	0.
10433.18	37823.27	0.
10433.18	42963.31	0.
10433.18	48611.86	0.
10433.18	54825.26	0.
10433.18	61660.00	0.
10433.18	69178.22	0.
10433.18	77448.26	0.
10433.18	86545.31	0.
10433.18	96552.05	0.
10433.18	107559.48	0.
10433.18	119667.64	0.
10433.18	132986.63	0.
10433.18	147637.51	0.
10433.18	163753.47	0.
10433.18	181481.04	0.
10433.18	200981.36	0.
12828.72	0.00	5.414E-06
12828.72	1352.22	1.787E-06
12828.72	2839.66	1.656E-06
12828.72	4475.84	1.438E-06
12828.72	6275.64	0.
12828.72	8255.42	0.
12828.72	10433.18	0.
12828.72	12828.72	0.

12828.72	15463.81	0.
12828.72	18362.41	0.
12828.72	21550.87	0.
12828.72	25058.17	0.
12828.72	28916.21	0.
12828.72	33160.04	0.
12828.72	37828.27	0.
12828.72	42963.31	0.
12828.72	48611.86	0.
12828.72	54825.26	0.
12828.72	61660.00	0.
12828.72	69178.22	0.
12828.72	77448.26	0.
12828.72	86545.31	0.
12828.72	96552.05	0.
12828.72	107559.48	0.
12828.72	119667.64	0.
12828.72	132986.63	0.
12828.72	147637.51	0.
12828.72	163753.47	0.
12828.72	181481.04	0.
12828.72	200981.36	0.
15463.81	0.00	4.296E-06
15463.81	1352.22	4.201E-06
15463.81	2839.66	3.394E-06
15463.81	4475.84	3.363E-06
15463.81	6275.64	8.341E-07
15463.81	8255.42	0.
15463.81	10433.16	0.
15463.81	12828.72	0.
15463.81	15463.81	0.
15463.81	18362.41	0.
15463.81	21550.87	0.
15463.81	25058.17	0.
15463.81	28916.21	0.
15463.81	33160.04	0.
15463.81	37828.27	0.
15463.81	42963.31	0.
15463.81	48611.86	0.
15463.81	54825.26	0.
15463.81	61660.00	0.
15463.81	69178.22	0.
15463.81	77448.26	0.
15463.81	86545.31	0.
15463.81	96552.05	0.
15463.81	107559.48	0.
15463.81	119667.64	0.
15463.81	132986.63	0.
15463.81	147637.51	0.
15463.81	163753.47	0.
15463.81	181481.04	0.
15463.81	200981.36	0.

ISOINTENSITY LINE FOR 3.16E-06 /CM2/SR IS NEAR (1.04E+04, 2.54E+03)

18362.41	0.00	7.402E-06
18362.41	1352.22	7.262E-06
18362.41	2839.66	6.907E-06

18362.41	4475.84	2.539E-06
18362.41	6275.64	2.154E-06
18362.41	8255.42	5.226E-07
18362.41	10433.18	0.
18362.41	12828.72	0.
18362.41	15463.81	0.
18362.41	18362.41	0.
18362.41	21550.87	0.
18362.41	25058.17	0.
18362.41	28916.21	0.
18362.41	33160.04	0.
18362.41	37828.27	0.
18362.41	42963.31	0.
18362.41	48611.86	0.
18362.41	54825.26	0.
18362.41	61660.00	0.
18362.41	69178.22	0.
18362.41	77448.26	0.
18362.41	86545.31	0.
18362.41	96552.05	0.
18362.41	107559.48	0.
18362.41	119667.64	0.
18362.41	132986.63	0.
18362.41	147637.51	0.
18362.41	163753.47	0.
18362.41	181481.04	0.
18362.41	200981.36	0.
21550.87	0.00	1.375E-05
21550.87	1352.22	1.354E-05
21550.87	2837.66	1.285E-05
21550.87	4475.84	4.657E-06
21550.87	6275.64	1.587E-06
21550.87	8255.42	1.328E-06
21550.87	10433.18	3.063E-07
21550.87	12828.72	0.
21550.87	15463.81	0.
21550.87	18362.41	0.
21550.87	21550.87	0.
21550.87	25058.17	0.
21550.87	28916.21	0.
21550.87	33160.04	0.
21550.87	37828.27	0.
21550.87	42963.31	0.
21550.87	48611.86	0.
21550.87	54825.26	0.
21550.87	61660.00	0.
21550.87	69178.22	0.
21550.87	77448.26	0.
21550.87	86545.31	0.
21550.87	96552.05	0.
21550.87	107559.48	0.
21550.87	119667.64	0.
21550.87	132986.63	0.
21550.87	147637.51	0.
21550.87	163753.47	0.
21550.87	181481.04	0.

21550.87	200981.36	0.

ISOINTENSITY LINE FOR 1.00E-05W/CM2/SR IS NEAR (1.55E+04, 2.84E+03)

25098.17	0.00	2.059E-05
25098.17	1352.22	2.034E-05
25098.17	2839.66	9.325E-06
25098.17	4475.84	8.653E-06
25098.17	6275.64	3.022E-06
25098.17	8255.42	1.094E-06
25098.17	10433.18	8.230E-07
25098.17	12828.72	1.996E-07
25098.17	15463.81	0.
25098.17	18362.41	0.
25098.17	21550.87	0.
25098.17	25058.17	0.
25098.17	28916.21	0.
25098.17	33160.04	0.
25098.17	37828.27	0.
25098.17	42963.31	0.
25098.17	48611.86	0.
25098.17	54825.26	0.
25098.17	61660.00	0.
25098.17	69178.22	0.
25098.17	77448.26	0.
25098.17	86545.31	0.
25098.17	96552.05	0.
25098.17	107559.48	0.
25098.17	119667.64	0.
25098.17	132986.63	0.
25098.17	147637.51	0.
25098.17	163753.47	0.
25098.17	181481.04	0.
25098.17	200981.36	0.
28916.21	0.00	2.029E-05
28916.21	1352.22	2.011E-05
28916.21	2839.66	1.404E-05
28916.21	4475.84	1.325E-05
28916.21	6275.64	5.342E-06
28916.21	8255.42	2.195E-06
28916.21	10433.18	7.351E-07
28916.21	12828.72	6.103E-07
28916.21	15463.81	1.298E-07
28916.21	18362.41	0.
28916.21	21550.87	0.
28916.21	25058.17	0.
28916.21	28916.21	0.
28916.21	33160.04	0.
28916.21	37828.27	0.
28916.21	42963.31	0.
28916.21	48611.86	0.
28916.21	54825.26	0.
28916.21	61660.00	0.
28916.21	69178.22	0.
28916.21	77448.26	0.
28916.21	86545.31	0.
28916.21	96552.05	0.
28916.21	107559.48	0.

28916.21	119667.64	0.
28916.21	132986.63	0.
28916.21	147637.51	0.
28916.21	163753.47	0.
28916.21	181481.04	0.
28916.21	200981.36	0.
33160.04	0.00	1.953E-05
33160.04	1352.22	1.843E-05
33160.04	2839.66	1.465E-05
33160.04	4475.84	1.024E-05
33160.04	6275.64	9.528E-06
33160.04	8255.42	4.082E-06
33160.04	10433.18	1.501E-06
33160.04	12828.72	4.395E-07
33160.04	15463.81	4.056E-07
33160.04	18362.41	6.397E-08
33160.04	21550.87	0.
33160.04	25058.17	0.
33160.04	28916.21	0.
33160.04	33160.04	0.
33160.04	37828.27	0.
33160.04	42963.31	0.
33160.04	48611.86	0.
33160.04	54825.26	0.
33160.04	61660.00	0.
33160.04	69178.22	0.
33160.04	77448.26	0.
33160.04	86545.31	0.
33160.04	96552.05	0.
33160.04	107559.48	0.
33160.04	119667.64	0.
33160.04	132986.63	0.
33160.04	147637.51	0.
33160.04	163753.47	0.
33160.04	181481.04	0.
33160.04	200981.36	0.
37828.27	0.00	1.448E-05
37828.27	1352.22	1.442E-05
37828.27	2839.66	1.420E-05
37828.27	4475.84	1.380E-05
37828.27	6275.64	7.219E-06
37828.27	8255.42	6.651E-06
37828.27	10433.18	3.061E-06
37828.27	12828.72	1.318E-06
37828.27	15463.81	4.204E-07
37828.27	18362.41	2.675E-07
37828.27	21550.87	8.083E-08
37828.27	25058.17	0.
37828.27	28916.21	0.
37828.27	33160.04	0.
37828.27	37828.27	0.
37828.27	42963.31	0.
37828.27	48611.86	0.
37828.27	54825.26	0.
37828.27	61660.00	0.
37828.27	69178.22	0.

37628.27	77448.26	0.
37828.27	86545.31	0.
37828.27	96552.05	0.
37628.27	107559.48	0.
37828.27	119667.64	0.
37828.27	132986.63	0.
37828.27	147637.51	0.
37828.27	163753.47	0.
37828.27	181481.04	0.
37828.27	200981.36	0.
42963.31	0.00	1.184E-05
42963.31	1352.22	1.180E-05
42963.31	2839.66	1.164E-05
42963.31	4475.84	1.138E-05
42963.31	6275.64	9.759E-06
42963.31	8255.42	5.650E-06
42963.31	10433.18	2.347E-06
42963.31	12828.72	2.088E-06
42963.31	15463.81	6.795E-07
42963.31	18362.41	2.795E-07
42963.31	21551.87	6.185E-08
42963.31	25059.17	5.190E-08
42963.31	28916.21	0.
42963.31	33160.04	0.
42963.31	37828.27	0.
42963.31	42963.31	0.
42963.31	48611.86	0.
42963.31	54825.26	0.
42963.31	61660.00	0.
42963.31	69178.22	0.
42963.31	77448.26	0.
42963.31	86545.31	0.
42963.31	96552.05	0.
42963.31	107559.48	0.
42963.31	119667.64	0.
42963.31	132986.63	0.
42963.31	147637.51	0.
42963.31	163753.47	0.
42963.31	181481.04	0.
42963.31	200981.36	0.
48611.86	0.00	9.961E-06
48611.86	1352.22	9.935E-06
48611.86	2839.66	9.173E-06
48611.86	4475.84	9.337E-06
48611.86	6275.64	8.320E-06
48611.86	8255.42	6.462E-06
48611.86	10433.18	3.893E-06
48611.86	12828.72	2.027E-06
48611.86	15463.81	6.529E-07
48611.86	18362.41	4.466E-07
48611.86	21551.87	1.816E-07
48611.86	25059.17	3.932E-08
48611.86	28916.21	3.214E-08
48611.86	33160.04	0.
48611.86	37828.27	0.
48611.86	42963.31	0.

48611.86	48611.86	0.
48611.86	54825.26	0.
48611.86	61660.00	0.
48611.86	69178.22	0.
48611.86	77448.26	0.
48611.86	86545.31	0.
48611.86	96552.05	0.
48611.86	107559.48	0.
48611.86	119667.64	0.
48611.86	132966.63	0.
48611.86	147637.51	0.
48611.86	163753.47	0.
48611.86	181461.04	0.
48611.86	200981.36	0.
54825.26	0.00	7.088E-06
54825.26	1352.22	7.073E-06
54825.26	2839.66	7.024E-06
54825.26	4475.64	6.904E-06
54825.26	6275.64	6.756E-06
54825.26	8255.42	6.377E-06
54825.26	10433.16	4.445E-06
54825.26	12828.72	2.648E-06
54825.26	15463.81	1.379E-06
54825.26	18362.41	4.422E-07
54825.26	21550.87	2.944E-07
54825.26	25058.17	1.205E-07
54825.26	28916.21	2.647E-08
54825.26	33160.04	2.230E-08
54825.26	37828.27	0.
54825.26	42963.31	0.
54825.26	48611.86	0.
54825.26	54825.26	0.
54825.26	61660.00	0.
54825.26	69178.22	0.
54825.26	77448.26	0.
54825.26	86545.31	0.
54825.26	96552.05	0.
54825.26	107559.48	0.
54825.26	119667.64	0.
54825.26	132966.63	0.
54825.26	147637.51	0.
54825.26	163753.47	0.
54825.26	181461.04	0.
54825.26	200981.36	0.
61660.00	0.00	5.301E-06
61660.00	1352.22	5.292E-06
61660.00	2839.66	5.262E-06
61660.00	4475.64	5.205E-06
61660.00	6275.64	5.115E-06
61660.00	8255.42	4.964E-06
61660.00	10433.16	4.769E-06
61660.00	12828.72	2.908E-06
61660.00	15463.81	2.323E-06
61660.00	18362.41	9.919E-07
61660.00	21550.87	3.252E-07
61660.00	25058.17	1.745E-07

61660.00	28916.21	8.491E-08
61660.00	33160.04	1.834E-08
61660.00	37828.27	1.547E-08
61660.00	42963.31	0.
61660.00	48611.86	0.
61660.00	54825.26	0.
61660.00	61660.00	0.
61660.00	69178.22	0.
61660.00	77448.26	0.
61660.00	86545.31	0.
61660.00	96552.05	0.
61660.00	107559.48	0.
61660.00	119667.64	0.
61660.00	132986.63	0.
61660.00	147637.51	0.
61660.00	163753.47	0.
61660.00	181481.04	0.
61660.00	200981.36	0.
69178.22	0.00	3.954E-06
69178.22	1352.22	3.949E-06
69178.22	2839.66	3.931E-06
69178.22	4475.84	3.897E-06
69178.22	6275.64	3.842E-06
69178.22	8255.42	3.763E-06
69178.22	10433.18	3.656E-06
69178.22	12828.72	3.357E-06
69178.22	15463.81	2.146E-06
69178.22	18362.41	1.151E-06
69178.22	21550.87	7.030E-07
69178.22	25058.17	2.310E-07
69178.22	28916.21	1.247E-07
69178.22	33160.04	7.015E-08
69178.22	37828.27	1.258E-08
69178.22	42963.31	1.061E-08
69178.22	48611.86	0.
69178.22	54825.26	0.
69178.22	61660.00	0.
69178.22	69178.22	0.
69178.22	77448.26	0.
69178.22	86545.31	0.
69178.22	96552.05	0.
69178.22	107559.48	0.
69178.22	119667.64	0.
69178.22	132986.63	0.
69178.22	147637.51	0.
69178.22	163753.47	0.
69178.22	181481.04	0.
69178.22	200981.36	0.
77448.26	0.00	3.019E-06
77448.26	1352.22	3.016E-06
77448.26	2839.66	3.005E-06
77448.26	4475.84	2.964E-06
77448.26	6275.64	2.948E-06
77448.26	8255.42	2.899E-06
77448.26	1.433.18	2.331E-06
77448.26	12828.72	2.715E-06

77448.26	15463.81	2.270E-06
77448.26	18362.41	1.545E-06
77448.26	21550.87	8.270E-07
77448.26	25058.17	3.323E-07
77448.26	28916.21	1.831E-07
77448.26	33160.04	5.700E-08
77448.26	37829.27	4.188E-08
77448.26	42963.31	1.341E-08
77448.26	48611.86	7.142E-09
77448.26	54825.26	0.
77448.26	61660.00	0.
77448.26	69179.22	0.
77448.26	77448.26	0.
77448.26	86545.31	0.
77448.26	96552.05	0.
77448.26	107559.48	0.
77448.26	119667.64	0.
77448.26	132986.63	0.
77448.26	147637.51	0.
77448.26	163753.47	0.
77448.26	181481.04	0.
77448.26	200981.36	0.
86545.31	0.00	2.379E-06
86545.31	1352.22	2.377E-06
86545.31	2839.66	2.370E-06
86545.31	4475.84	2.356E-06
86545.31	6273.64	2.335E-06
86545.31	8255.42	2.303E-06
86545.31	10433.18	2.259E-06
86545.31	12828.72	2.202E-06
86545.31	15463.81	1.973E-06
86545.31	18362.41	1.647E-06
86545.31	21550.87	1.146E-06
86545.31	25058.17	6.321E-07
86545.31	28916.21	2.365E-07
86545.31	33160.04	1.285E-07
86545.31	37829.27	5.356E-08
86545.31	42963.31	3.350E-08
86545.31	48611.86	9.185E-09
86545.31	54825.26	4.749E-09
86545.31	61660.00	0.
86545.31	69179.22	0.
86545.31	77448.26	0.
86545.31	86545.31	0.
86545.31	96552.05	0.
86545.31	107559.48	0.
86545.31	119667.64	0.
86545.31	132986.63	0.
86545.31	147637.51	0.
86545.31	163753.47	0.
86545.31	181481.04	0.
86545.31	200981.36	0.
96552.05	0.00	2.032E-06
96552.05	1352.22	2.030E-06
96552.05	2839.66	2.025E-06
96552.05	4475.84	2.016E-06

96552.05	6275.64	2.001E-06
96552.05	8255.42	1.728E-06
96552.05	10433.18	1.698E-06
96552.05	12828.72	1.358E-06
96552.05	15463.81	1.570E-06
96552.05	18362.41	1.342E-06
96552.05	21550.87	1.032E-06
96552.05	25053.17	5.364E-07
96552.05	28916.21	4.435E-07
96552.05	33160.04	1.751E-07
96552.05	37828.27	9.395E-08
96552.05	42963.31	3.799E-08
96552.05	48611.86	2.232E-08
96552.05	54825.26	6.056E-09
96552.05	61660.00	1.130E-09
96552.05	69178.22	0.
96552.05	77448.26	0.
96552.05	86545.31	0.
96552.05	96552.05	0.
96552.05	107559.46	0.
96552.05	119667.64	0.
96552.05	132986.63	0.
96552.05	147637.51	0.
96552.05	163753.47	0.
96552.05	181451.04	0.
ISCINTENSITY LINE FOR 1.00E-06W/CM2/SR IS NEAR (7.74E+04, 2.16E+04)		
107559.48	0.00	1.757E-06
107559.48	1352.22	1.756E-06
107559.48	2839.66	1.753E-06
107559.48	4475.84	1.747E-06
107559.48	6275.64	1.737E-06
107559.48	8255.42	1.722E-06
107559.48	10433.18	1.702E-06
107559.48	12828.72	1.539E-06
107559.48	15463.81	1.465E-06
107559.48	18362.41	1.281E-06
107559.48	21550.87	1.101E-06
107559.48	25053.17	7.574E-07
107559.48	28916.21	4.270E-07
107559.48	33160.04	2.244E-07
107559.48	37828.27	1.382E-07
107559.48	42963.31	6.274E-08
107559.48	48611.86	2.462E-08
107559.48	54825.26	9.074E-09
107559.48	61660.00	4.106E-09
107559.48	69178.22	2.363E-09
107559.48	77448.26	0.
107559.48	86545.31	0.
107559.48	96552.05	0.
107559.48	107559.48	0.
107559.48	119667.64	0.
107559.48	132986.63	0.
107559.48	147637.51	0.
107559.48	163753.47	0.
107559.48	181481.04	0.
119667.64	0.00	1.102E-06

119667.64	1352.22	1.401E-06
119667.64	2839.66	1.399E-06
119667.64	4475.84	1.395E-06
119667.64	6275.64	1.388E-06
119667.64	8255.42	1.378E-06
119667.64	10433.18	1.365E-06
119667.64	12828.72	1.346E-06
119667.64	15463.81	1.322E-06
119667.64	18362.41	1.153E-06
119667.64	21550.67	9.965E-07
119667.64	25058.17	8.503E-07
119667.64	28916.21	5.257E-07
119667.64	33160.04	3.203E-07
119667.64	37828.27	1.684E-07
119667.64	42963.31	7.387E-08
119667.64	48611.86	4.806E-08
119667.64	54825.26	1.972E-08
119667.64	61660.00	1.140E-08
119667.64	69178.22	3.434E-09
119667.64	77448.26	1.454E-09
119667.64	86545.31	0.
119667.64	96552.05	0.
119667.64	107559.48	0.
119667.64	119667.64	0.
119667.64	132986.63	0.
119667.64	147637.51	0.
119667.64	163753.47	0.
119667.64	181461.04	0.
132986.63	0.00	1.155E-06
132986.63	1352.22	1.154E-06
132986.63	2839.66	1.153E-06
132986.63	4475.84	1.150E-06
132986.63	6275.64	1.103E-06
132986.63	8255.42	1.097E-06
132986.63	10433.18	1.087E-06
132986.63	12828.72	1.075E-06
132986.63	15463.81	1.047E-06
132986.63	18362.41	9.475E-07
132986.63	21550.67	9.220E-07
132986.63	25058.17	7.614E-07
132986.63	28916.21	7.294E-07
132986.63	33160.04	4.111E-07
132986.63	37828.27	3.237E-07
132986.63	42963.31	1.336E-07
132986.63	48611.86	6.254E-08
132986.63	54825.26	3.449E-08
132986.63	61660.00	1.575E-08
132986.63	69178.22	8.755E-09
132986.63	77448.26	2.362E-09
132986.63	86545.31	9.889E-10
132986.63	96552.05	0.
132986.63	107559.48	0.
132986.63	119667.64	0.
132986.63	132986.63	0.
132986.63	147637.51	0.
132986.63	163753.47	0.

[A] have only $s < 8$ significant digits, the user may compute sigrev, the corrected number of significant digits in the inverse by the formula

$$\text{sigrev} = \text{sigdig} - 8.3 + s$$

To get some idea how accurate sigrev itself was, the difference between sigrev and sigact, the actual average number of significant digits, was computed for many matrices of random numbers. When alsmo all the elements of the matrix were of the same order of magnitude, the results (for a batch of 120 matrices) were:

$$-1.17 < (\text{ sigrev } - \text{ sigact }) < 0.75$$

However, when the elements of the matrix ranged over three orders of magnitude, the results for a batch of 160 matrices were:

$$-1.77 < (\text{ sigrev } - \text{ sigact }) < 0.78$$

Both batches contained 100 5 by 5 matrices, the other matrices all being 10 by 10.

A DOUBLE PRECISION substitute for this subprogram is available.

I. Error Indications - ierror is an INTEGER variable which is used to flag errors.

ierror = -1 indicates both of the following:

1. Neither $[A]^{-1}$ nor [X] could be computed.
2. [A] was either singular* or very nearly so (usually the former.

   ierror = 1 indicates that no errors were detected.

   Due to round-off errors, a singular matrix will rarely be flagged as singular (by ierror = -1) unless it has a row or column of zeros. However, the value of sigdig will seldom be greater than 1.5 for singular matrices.

Division by zero cannot occur in this subprogram.

No test for floating-point overflow is made in this subprogram.

---

*When [A] is singular, a system of simultaneous equations [A][X] = [B] will have a solution if and only if [B] can be expressed as a linear combination of the columns of [A].

181481.04	0.00	5.972E-07
181481.04	1352.22	5.271E-07
181481.04	2839.66	5.966E-07
181481.04	4475.84	5.957E-07
181481.04	6275.64	5.943E-07
181481.04	8255.42	5.923E-07
181481.04	10433.18	5.894E-07
181481.04	12828.72	5.654E-07
181481.04	15463.81	5.643E-07
181481.04	18362.41	5.222E-07
181481.04	21550.67	4.987E-07
181481.04	25058.17	4.891E-07
181481.04	28916.21	4.772E-07
181481.04	33160.04	4.317E-07
181481.04	37828.27	3.595E-07
181481.04	42963.31	2.781E-07
181481.04	48611.86	1.790E-07
181481.04	54825.26	8.360E-08
181481.04	61660.00	6.548E-08
181481.04	69178.22	2.416E-08
181481.04	77448.26	1.375E-08
181481.04	86545.31	6.644E-09
181481.04	96552.05	2.355E-09
181481.04	107559.48	1.124E-09
181481.04	119667.64	1.646E-10

ISOINTENSITY LINE FOR 3.16E-07W/CM2/SR IS NEAR (1.48E+05, 3.78E+04)

200981.36	0.00	4.611E-07
200981.36	1352.22	4.611E-07
200981.36	2839.66	4.608E-07
200981.36	4475.84	4.602E-07
200981.36	6275.64	4.593E-07
200981.36	8255.42	4.580E-07
200981.36	10433.18	4.561E-07
200981.36	12828.72	4.535E-07
200981.36	15463.81	4.408E-07
200981.36	18362.41	4.364E-07
200981.36	21550.87	4.300E-07
200981.36	25058.17	4.132E-07
200981.36	28916.21	3.939E-07
200981.36	33160.04	3.715E-07
200981.36	37828.27	3.125E-07
200981.36	42963.31	2.528E-07
200981.36	48611.86	2.363E-07
200981.36	54825.26	1.361E-07
200981.36	61660.00	5.377E-08
200981.36	69178.22	4.337E-08
200981.36	77448.26	1.584E-08
200981.36	86545.31	5.872E-09
222431.71	0.00	0.

ERROR SUMMARY

ERROR	TIMES
0049	0.02

Appendix C
AUXILIARY LIBRARY ROUTINES

Operating descriptions and listings are presented here for LSQPØL and SID--two subroutines from the MDAC Library. These were not programmed under this contract, but are used by the FLAME code.

C.1 LSQPØL

Least Squares Polynomial SUBROUTINE

A. Description - This subroutine subprogram computes the coefficients of the polynomial which best fits (in the least squares sense) a given function of one independent variable.

B. Use - CALL LSQPOL (x, y, w, r, n, s, m, c) where:

- x is the REAL array, dimensioned at least n, of the values of the independent variable of the given function.
- y is the REAL array, dimensioned at least n, of the values of the dependent variable of the given function.
- w is the REAL array, dimensioned at least n, of the least squares weighting for each point of the given function. (If an unweighted least squares fit is desired, each element of this array must be a 1.)
- r is the REAL array, dimensioned at least n, which this subprogram sets equal to the residuals at each point of the given function.
- n is an INTEGER variable (or constant) denoting the number of points defining the given function. This number must be ≤ 50.
- s is the REAL variable which this subprogram sets equal to the sum of the squares of the residuals.
- m is the INTEGER variable (or constant) denoting how many coefficients the least square polynomial will have. That is, it is the degree plus one of the least squares polynomial desired. This number must be input ≤ 10.

c is the REAL array, dimensioned at least m whose elements are set equal to the coefficients of the least squares polynomial beginning with the constant term.

C. Support Level - Supported by Programming Systems and Support Branch of Information Processing Systems.

D. Language Used - FORTRAN.

E. Availability - On FORTRAN library.

F. Extent - 4052_8 words.

G Timing - Not available.

H. Restrictions - None.

I. Error Indications - The least squares problem is mathematically inconsistent unless $m \leq n$, therefore, if $m > n$ the following note is printed and execution of the program is terminated.

LSQPOL ERROR — MORE COEFFICIENTS THAN POINTS

J. Method - The method used is completely described in "Polynomial Curve Fitting With Constraint" by J. E. L. Peck in SIAM Review, volume 4, number 2, April, 1962.

This subprogram finds the c_i's such that:

$$s = \sum_{k=1}^{n} w_k r_k^2 \text{ is a minimum.}$$

$$r_k = y_k - \sum_{i=1}^{m} c_i x_k^{i-1}$$

K. Examples - None.

C.2 SID

Single Precision Simultaneous Equation Solution and Matrix Inversion SUBROUTINE

A. Description - This subroutine subprogram will invert a REAL nonsingular square matrix, [A], and if desired, also solve linear system(s) of simultaneous equations, [A] [X] = [B], where [B] may have any number of columns. Every column of [B] must have at least one; non-zero element. The dimensions of [A] and [B] are limited only by the available core storage.

B. Use - CALL SID (amat, nrow, maxrow, mxcola, bmat, ncolb, mxcolb, sigdig, ierror, rtemp, itemp, scaleb) where:

1. Input

amat	is a REAL two dimensional array which contains [A]. [A] will be replaced by $[A]^{-1}$ during the execution of this subprogram.
nrow	is an INTEGER variable or constant which denotes the number of rows in [A]. nrow may be as small as one (1) or as large as available core storage will permit.
maxrow	is an INTEGER variable or constant which denotes the maximum number of rows which may be stored in the amat array. Note that a matrix may have fewer rows and/or columns than the array which contains it.
mxcola	is an INTEGER variable or constant which denotes the maximum number of columns which may be stored in the amat array.
bmat	is a Real array which contains [B] if [B] is present. If [B] is present, the bmat array must have exactly maxrow rows. [B] will be replaced by [X] during the execution of this subprogram. If no [B] is present, bmat may be any variable or constant of any type.
ncolb	is an INTEGER variable or constant which denotes the number of columns in [B]. If there is no [B], use ncolb = 0. ncolb may be as small as zero or as large as available core storage will permit.
mxcolb	is an INTEGER variable or constant which denotes the maximum number of columns which may be stored in the bmat array. If no [B] is present, mxcolb may have any value.

2. Output

amat	will contain $[A]^{-1}$.
bmat	will contain [X] if [B] was present.

sigdig — is a REAL variable which will be set equal to an estimate of the number of significant digits in the elements of the inverse. However, this estimate is based on the assumption that the elements of [A] are all accurate to eight significant digits. The Restrictions section describes a simple adjustment of sigdig which the user must make when the elements of [A] have fewer than eight significant digits.

ierror — is an INTEGER variable which is used to flag errors. Its meaning is explained in Error Indications.

3. Intermediate - The following three arrays are used during calculation for the temporary storage of intermediate results. It makes no difference whether the arrays are one, two, or three dimensional. The three arrays need not all be of the same dimension.

rtemp — is a REAL array which contains at least 3·nrow elements.

itemp — is an INTEGER array which has at least 3·nrow elements.

scaleb — is a REAL array which contains at least ncolb elements.

C. Support Level - Supported by Programming Systems and Support Branch of Information Processing Systems.

D. Language Used - MSSD Standard FORTRAN.

E. Availability - On the FORTRAN library.

F. Extent - 766 words, 19 words for ALOG10, and 46 words for ALOG.

G. Timing - Not available.

H. Restrictions - This subprogram cannot find a solution to the system of simultaneous equations [A] [X] = [0].

An estimate of the average number of significant digits in the elements of the inverse is given by the argument sigdig. (See the Use section.) However, this estimate is only valid when the elements of A have eight significant digits. When the elements of

[A] have only s < 8 significant digits, the user may compute sigrev, the corrected number of significant digits in the inverse by the formula

$$\text{sigrev} = \text{sigdig} - 8.3 + s$$

To get some idea how accurate sigrev itself was, the difference between sigrev and sigact, the actual average number of significant digits, was computed for many matrices of random numbers. When alsmo all the elements of the matrix were of the same order of magnitude, the results (for a batch of 120 matrices) were:

$$-1.17 < (\text{sigrev} - \text{sigact}) < 0.75$$

However, when the elements of the matrix ranged over three orders of magnitude, the results for a batch of 160 matrices were:

$$-1.77 < (\text{sigrev} - \text{sigact}) < 0.78$$

Both batches contained 100 5 by 5 matrices, the other matrices all being 10 by 10.

A DOUBLE PRECISION substitute for this subprogram is available.

I. Error Indications - ierror is an INTEGER variable which is used to flag errors.

ierror = -1 indicates both of the following:

1. Neither $[A]^{-1}$ nor [X] could be computed.
2. [A] was either singular* or very nearly so (usually the former.

   ierror = 1 indicates that no errors were detected.

   Due to round-off errors, a singular matrix will rarely be flagged as singular (by ierror = -1) unless it has a row or column of zeros. However, the value of sigdig will seldom be greater than 1.5 for singular matrices.

Division by zero cannot occur in this subprogram.

No test for floating-point overflow is made in this subprogram.

*When [A] is singular, a system of simultaneous equations [A][X] = [B] will have a solution if and only if [B] can be expressed as a linear combination of the columns of [A].

J. Method - The numerical method used is basically the Gauss-Jordan elimination with selection of maximum pivotal elements (full pivoting). A description of the Gauss-Jordan method is contained in Numerical Analysis by Kaiser S. Kunz; McGraw-Hill, 1957.

Special procedures are present to improve accuracy and also to minimize the frequency of overflow and underflow when the magnitudes of any of the elements in the A or B matrices are either large or small.

K. Example - The coding below will invert a matrix, solve a system of simultaneous equations, and print a warning note if the inverse has less than about one significant digit. The elements of the matrix originally in AMAT are assumed to have an average accuracy of eight significant digits.

```
      .
      .
      .
  DIMENSIØN AMAT (10,10), BMAT(10,1 ), RTEMP(10,3),
 ITEMP(10,3), 1   SCALEB(1)
      .
      .
      .
 NRØW = 6
      .
      .
      .
  CALL SID (AMAT,NRØW, 10, 10, BMAT, 1, 1, SIGDIG, IERRØR,
 1   RTEMP, ITEMP, SCALEB)
  IF (SIGDIG .LT. 2.) WRITE (6, 180) SIGDIG
180 FØRMAT (40H1WARNING - MATRIX INVERSE HAS ØNLY ABØUT,
 1   F5.1, 20H SIGNIFICANT DIGITS.)
```

```
      SUBROUTINE SID (A, N, NDROW, NDCOLA, B, M, NDCOLB, SIGDIG, IERROR,
     .   PIVOT, INDEX, SCALEB )
C
C     SID - A SINGLE PRECISION SIMULTANEOUS EQUATION SOLVER, INVERSE
C                    FINDER, AND DETERMINANT SUBROUTINE
C
      DIMENSION  A(NDROW,NDCOLA), B(NDROW,NDCOLB), PIVOT(N,3),
     .   SCALEB(M), INDEX(N,3)
      DOUBLE PRECISION  DBIGP2
      DATA         DBIGP2
     . /  73786976294838?9.D4
C
      EPS = 1.E-3
  712 EPS = EPS/2.
      EPSP15= EPS + 1.5
      IF (EPSP15 .NE. 1.5)  GO TO 712
      SIGMCH = ALOG10(1.522/EPS)
      BIGPW2 = DBIGP2
      PIVOT(1,1) = 0.
C
C     SCALE ROWS
C
      DO 38 I=1,N
      ROWMX = 0.
      DO 28 J=1,N
      IF ((ABS(A(I,J))) .GT. ROWMX)  ROWMX =  ABS(A(I,J))
   28 CONTINUE
      IF ( ROWMX)  29, 750, 29
   29 CONTINUE
      ROWMXI = 1. / ROWMX
      DO 32 J=1,N
      AIJ = A(I,J)
      A(I,J) = (A(I,J) * ROWMXI ) * BIGPW2
      IF (A(I,J) .EQ. 0.)  A(I,J) = (AIJ * BIGPW2) * ROWMXI
   32 CONTINUE
      IF (M) 34, 38, 34
   34 DO 36 J=1,M
      BIJ = B(I,J)
      B(I,J) = (B(I,J) * ROWMXI ) * BIGPW2
      IF (B(I,J) .EQ. 0.)  B(I,J) = (BIJ * BIGPW2) * ROWMXI
   36 CONTINUE
   38 PIVOT(I,2) = ROWMXI
C
C     SCALE COLUMNS
C
      DO 10 J=1,N
      COLMX = 0.
      DO 4 I=1,N
      IF (ABS(A(I,J)).GT. COLMX) COLMX = ABS(A(I,J))
    4 CONTINUE
      IF ( COLMX )  5, 750, 5
    5 CONTINUE
      COLMXI = 1./COLMX
      DO 8 I=1,N
      AIJ = A(I,J)
      A(I,J) = (A(I,J) * COLMXI)* BIGPW2
      IF (A(I,J) .EQ. 0.)  A(I,J) = (AIJ * BIGPW2) * COLMXI
    8 CONTINUE
   10 PIVOT(J,3) = BIGPW2 * COLMXI
```

```
      IF (M) 14,24,14
   14 DO 22 J=1,M
      COLMX = 0.
      DO 16 I=1,N
      IF (ABS(B(I,J)).GT. COLMX )  COLMX = ABS(B(I,J))
   16 CONTINUE
      IF (COLMX )  17, 22, 17
   17 CONTINUE
      SCALEB(J) = COLMX / BIGPW2
      COLMXI = 1./COLMX
      DO 20 I=1,N
      BIJ = B(I,J)
      B(I,J) = (B(I,J) * COLMXI) * BIGPW2
      IF (B(I,J) .EQ. 0.)  B(I,J) = (BIJ * BIGPW2) * COLMXI
   20 CONTINUE
   22 CONTINUE
   24 CONTINUE
C
C     INITIALIZATION
C
      PMONE=1.
      DO 42 J=1,N
      PIVOT(J,1) = 0.
   42 INDEX(J,3) = 0
      DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
      ABPIVI=0.
   45 DO 105 J=1,N
   50 IF (INDEX(J,3)-1) 60,105,60
   60 DO 100 K=1,N
   70 IF (INDEX(K,3)-1) 80,100,80
   80 IF (ABS(A(J,K)) - ABPIVI) 100,100,85
   85 IROW=J
   90 ICOLUM=K
      ABPIVI=ABS(A(J,K))
  100 CONTINUE
  105 CONTINUE
      IF (I-1) 115,120,115
  115 IF ( ABPIVI .GE. PIVMIN ) GO TO 123
  120 PIVMIN=ABPIVI
      IF (ABPIVI) 123,750,123
  123 CONTINUE
      INDEX(ICOLUM,3)=1
      PIVOTI=A(IROW,ICOLUM)
      PIVOT(I,1) = PIVOTI
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
  130 IF (IROW-ICOLUM) 140, 260, 140
  140 PMONE=-PMONE
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
  205 IF (M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP = B(IROW,L)
```

```
  230 B(IROW,L) = B(ICOLUM,L)
  250 B(ICOLUM,L) = SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
      PIVINV=1.0/PIVOTI
  330 A(ICOLUM,ICOLUM) = BIGPW2
  340 DO 350 L=1,N
  350 A(ICOLUM,L)= A(ICOLUM,L)*PIVINV
  355 IF (M) 380, 380, 360
  360 DO 370  L=1,M
  370 B(ICOLUM,L) = B(ICOLUM,L)*PIVINV
C
C     REDUCE NON-PIVOT ROWS
C
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
      IF (T) 420,550,420
  420 A(L1,ICOLUM)=0.0
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF(M) 550, 550, 460
  460 DO 500 L=1,M
  500 B(L1,L) = B(L1,L) - B(ICOLUM,L)*T
  550 CONTINUE
C
C     INTERCHANGE COLUMNS
C
  600 DO 710 I=1,N
  610 L=N+1-I
  620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
  640 JCOLUM=INDEX(L,2)
  650 DO 705 K=1,N
  660 SWAP=A(K,JROW)
  670 A(K,JROW)=A(K,JCOLUM)
  700 A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
C
      PIVOT(1,1) = PIVOT(1,1) * PMONE
C
      SIGDIG = SIGMCH - ALOG10(BIGPW2/PIVMIN)
      IF (SIGDIG .LT. .85)  SIGDIG = 0.
C
C     UNSCALE INVERSE AND SOLUTION(S)
C
      DO 720 J=1,N
      ROWMXI = PIVOT(J,2)
      DO 720 I=1,N
      IF (ROWMXI .LT. 1.) GO TO 715
      A(I,J) = (A(I,J) * PIVOT(I,3)) * ROWMXI
      GO TO 720
  715 A(I,J) = A(I,J) * (PIVOT(I,3) * ROWMXI)
  720 CONTINUE
      IF (M) 725, 735, 725
```

```
  725 DO 730 J=1,M
      ROWMXI = SCALER(J)
      DO 730 I=1,N
      IF (ROWMXI .LT. 1.) GO TO 728
      B(I,J) = (B(I,J) * PIVOT(I,3)) * ROWMXI
      GO TO 730
  728 B(I,J) = B(I,J) * (PIVOT(I,3) * ROWMXI)
  730 CONTINUE
  735 CONTINUE
C
      IERROR = 1
      RETURN
  750 IERROR = -1
      SIGDIG = 0.
      RETURN
      END
```

```
      SUBROUTINE LSQPOL (XI,YI,WI,RO,N,SUM,M,COEF)
      DIMENSION XI(1),YI(1),WI(1),RO(1),COEF(1)
      COMMON / Z/LSQP/
     .          X(50),Y(50),W(50),B(11),A(11),S(11),DSQ(11)
     .         ,P(50),PO(50),C(11)
      DOUBLE PRECISION  X,Y,W,B,A,S,PO,P,DSQ,C,G
      DO 100 I = 1,N
      X(I) = XI(I)
      Y(I) = YI(I)
  100 W(I) = WI(I)
      CALL ORPOLG(M-1,X,Y,W,N,B,A,S,DSQ,PO,P,G,E)
      IF(E) 1100,150,1100
  150 CALL ORPOLC(M-1,C,S,A,B,G,P,PO)
      DO 200 I = 1,M
  200 COEF(I) = C(I)
      SUM = 0.0
      DO 500 I = 1,N
      K = M-1
      Z = COEF(M)
      IF (K) 300,350,300
  300 Z = Z*XI(I) + COEF(K)
      K = K-1
      IF (K) 350,350,300
  350 RO(I) = Z - YI(I)
  500 SUM = SUM + RO(I)**2
      RETURN
 1100 WRITE (6,1200)
 1200 FORMAT(43H1LSQPOL ERROR-MORE COEFFICIENTS THAN POINTS)
      STOP
      END
```

```
      SUBROUTINE ORPOLG(N,X,Y,W,M,B,A,S,DSQ,PO,P,GAMMA,ERROR)
      DOUBLE PRECISION X,Y,W,B,A,PO,P ,DSQ,WPP,WXPP,WYP,TEMP
      DOUBLE PRECISION  WPPO,SUM,GAMMA,T                ,MEAN,S
      EQUIVALENCE (T,TEMP,SUM)
      DIMENSION X(1),Y(1),W(1),B(1),A(1),S(1),PO(1),P (1),
     1          DSQ(1)
      SUM=0.
      DO 6 I=1,M
    6 SUM=SUM+X(I)
      MEAN=SUM/FLOAT(M)
      T=0.
      DO 7 I=1,M
CD    T=AMAX1(T,ABS(X(I)-MEAN))
    7 TT = DABS(X(I) - MEAN)
      IF (TT .GT. T) T = TT
      GAMMA=T/2.
      DO 8 I=1,M
    8 X(I)=(X(I)-MEAN)/GAMMA
      ERROR=0.
      NO=M-N-1
  101 IF(NO)102,103,103
  102 ERROR=1.
      GO TO 105
  103 B=0.
      DSQ=0.
      WPP=0.
      DO 106 J=1,M
      P (J)=1.
      PO(J)=0.
      WPP=WPP+W(J)
      IF(NO)106,106,107
  107 DSQ=DSQ+W(J)*Y(J)**2
  106 CONTINUE
      NA=N+1
      DO 109 I=1,NA
      WXPP=0.
      WYP=0.
      DO 110 J=1,M
      TEMP=W(J)*P(J)
      IF(NA-I)111,111,112
  112 WXPP=WXPP+TEMP*X(J)*P(J)
  111 IF(M - I)110,113,113
  113 WYP=WYP+TEMP*Y(J)
  110 CONTINUE
      IF(M - I)114,115,115
  115 S(I)=WYP/WPP
  114 IF(NA-M)666,117,117
  666 IF(I-1)102,667,116
  667 DSQ=DSQ-S**2*WPP
      GO TO 117
  116 DSQ(I)=DSQ(I-1)-S(I)**2*WPP
  117 IF(NA-I)109,109,119
  119 A(I)=WXPP/WPP
      WPPO=WPP
      WPP=0.
      DO 120 J=1,M
      TEMP=(X(J)-A(I))*P(J)-B(I)*PO(J)
      WPP=WPP+W(J)*TEMP**2
```

```
      PO(J)=P(J)
120   P(J)=TEMP
      B(I+1)=WPP/WPPO
109   CONTINUE
      IF (N) 140,155,140
 140  DO 150 I=1,N
      A(I)=A(I)*GAMMA + MEAN
150   B(I)=B(I)*GAMMA
 155  DO 160 I=1,M
160    X(I)=GAMMA*X(I)+MEAN
 105   RETURN
      END
```

```
      SUBROUTINE ORPOLC (N,C,S,A,B,G,P,PM)
      DOUBLE PRECISION  C,S,A,B,G,P,PM,T1,T2
      DIMENSION C(1),S(1),A(1),B(1),P(1),PM(1)
      M=N+1
      DO 300 L=1,M
      C(L)=0.
      PM(L)=0.
  300 P(L)=0.
      P=1.
      C=S
      IF (N) 301,303,301
  301 DO 302 L=1,N
      T2=0.
      J=L+1
      DO 302 M=1,J
      T1=(T2-A(L)*P(M)-B(L)*PM(M))/G
      T2=P(M)
      PM(M)=P(M)
      P(M)=T1
  302 C(M)=C(M)+T1*S(L+1)
  303 RETURN
      END
```

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13. ABSTRACT

As part of the Optical Signatures Program, McDonnell Douglas Astronautics Company-West has developed the initial working model of a code to describe the gross features of rocket-plume radiation for altitudes above 75 n mi. The main effort is the construction of a scheme for integration of an arbitrary function through an arbitrary axisymmetric rocket plume, with any specified look angle, plume direction, and vehicle velocity direction. Radiances are presented as integrated values in a specified spectral band. The equations used and a printout of the code and of a sample application are included.

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